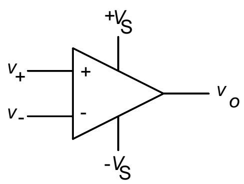
Operational Amplifiers

These are common components in conditioning circuits.

There are two inputs, v_+ and v_i, two power supplies, $+V_{\rm s}$ and $-V_{\rm s}$, and an output, v_o.



$v_o = \min\{+V_{\rm s}, \max\{-V_{\rm s}, (v_+ - v_-)A\}\}$, were A is the op-amp gain.

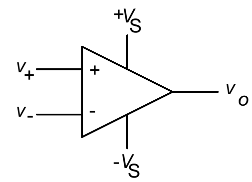
Ignoring saturation, $v_o = A\,(v_+ - v_-)$.

Ideal Op-Amp Properties

Infinite input impedance.

Infinite gain. $(A = \infty)$

Zero output impedance.



Where to Find Ideal Op-Amps

An electronics textbook.

However, in certain circuits a real op-amp performs almost the same as an ideal op-amp would.

Simplifying Assumptions

Current into inputs is zero.

When used in a negative feedback configuration, $v_+ = v_-$.

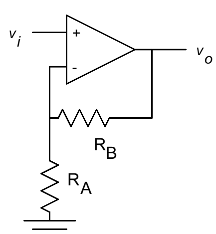
Op-Amp Circuits to be Covered

Non-inverting amplifier.

Inverting amplifier.

Summing amplifier.

Non-Inverting Amplifier



$v_o = \dfrac{R_{\rm A} + R_{\rm B}}{R_{\rm A}} v_i.$

Versatility of Inverting Amplifier

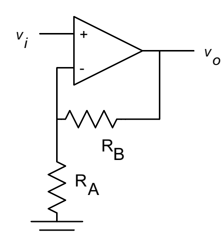
Use of Non-Inverting Amplifier in Conditioning Circuits

$v_o = \dfrac{R_{\rm A} + R_{\rm B}}{R_{\rm A}} v_i.$

Traditional Use, Voltage Amplifier

Input is v_i, output is v_o.

$H_{\rm c}(v_i) = \dfrac{R_{\rm A} + R_{\rm B}}{R_{\rm A}} v_i$



Non-Inverting Amplifier Example Problem

Design a system with output $v_o = H(x)$, where process variable x is water level, $x \in [0\,\mathrm{m}, 1\,\mathrm{m}]$, and $H(x) = 10x\frac{\mathrm{V}}{\mathrm{m}}$.

Note: most example problems will not be as complete as the archetypical problem covered earlier.

Solution:

Use same float-and-cable system as in previous example problem.

Use $100\,\mathrm{k}\Omega$ three-terminal variable resistor with $1\,\mathrm{V}$ voltage source across fixed terminals:

$H_\mathrm{t}(x) = 1x\frac{\mathrm{V}}{\mathrm{m}}$.

Problem will be solved two ways:

First way, we know what kind of conditioning circuit is needed.

Second way, we have to determine algebraicly the type of conditioning circuit needed.

First Way: Use Non-Inverting Amplifier

Obviously, all that is needed is an amplifier with a gain of 10. A non-inverting amplifier will do.

Then:

$$\begin{aligned} H(x) &= H_\mathrm{c}(H_\mathrm{t}(x)) = A\left(1x\frac{\mathrm{V}}{\mathrm{m}}\right) \\ 10x\frac{\mathrm{V}}{\mathrm{m}} &= A\left(1x\frac{\mathrm{V}}{\mathrm{m}}\right) \\ A &= 10 \end{aligned}$$

So choose resistors such that $(R_\mathrm{A} + R_\mathrm{B})/R_\mathrm{A} = 10$.

For example, $R_\mathrm{A} = 10\,\mathrm{k}\Omega$ and $R_\mathrm{B} = 90\,\mathrm{k}\Omega$.

Second Way: Derive Conditioning-Circuit Function

Pretend we don't know that a simple amplifier is needed.

$$H(x) = H_\mathrm{c}(H_\mathrm{t}(x))$$

We need to solve for H_c.

Let $y = H_\mathrm{t}(x) = 1x\frac{\mathrm{V}}{\mathrm{m}}$.

Then $x = H_\mathrm{t}^{-1}(y) = 1y\frac{\mathrm{m}}{\mathrm{V}}$.

Substituting:

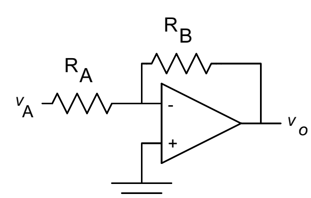
$$\begin{aligned} H\left(1y\frac{\mathrm{m}}{\mathrm{V}}\right) &= H_\mathrm{c}(y) \\ H_\mathrm{c}(y) &= H\left(1y\frac{\mathrm{m}}{\mathrm{V}}\right) = 10\left(1y\frac{\mathrm{m}}{\mathrm{V}}\right) \\ &= 10y \end{aligned}$$

Therefore our conditioning circuit needs to multiply y, a voltage, by a constant.

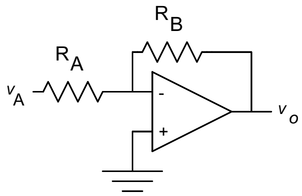
A non-inverting amplifier will do just that.

(The remainder of the solution is identical to the first way.)

Inverting Amplifier



$v_o = -\frac{R_\mathrm{B}}{R_\mathrm{A}}v_\mathrm{A}$.



Output $v_o = -\frac{R_\mathrm{B}}{R_\mathrm{A}} v_\mathrm{A}$.

Traditional Use, Voltage Amplifier

Input is v_A and output is v_o.

$H_\mathrm{c}(v_\mathrm{A}) = -\frac{R_\mathrm{B}}{R_\mathrm{A}} v_\mathrm{A} = A_1 v_\mathrm{A}$

where $A_1 = -\frac{R_\mathrm{B}}{R_\mathrm{A}}$.

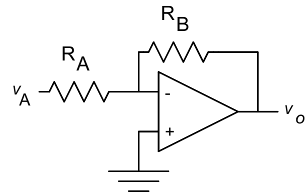
Resistance to Voltage Converter

Input is R_B, output is v_o.

$H_\mathrm{c}(R_\mathrm{B}) = -\frac{R_\mathrm{B}}{R_\mathrm{A}} v_\mathrm{A} = A_2 R_\mathrm{B}$

where $A_2 = -v_\mathrm{A}/R_\mathrm{A}$.

Here, v_A is a fixed voltage, buried in the constant A_2.



Inverted Resistance to Voltage Converter

Input is R_A, output is v_o.

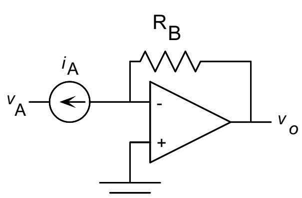
$H_\mathrm{c}(R_\mathrm{A}) = -\frac{R_\mathrm{B}}{R_\mathrm{A}} v_\mathrm{A} = A_3/R_\mathrm{A}$

where $A_3 = -R_\mathrm{B} v_\mathrm{A}$.

v_A is a fixed voltage here also.

Current to Voltage Converter

This circuit is similar to the inverting amplifier.



Input is i_A, output is v_o.

$H_\mathrm{c}(i_\mathrm{A}) = R_\mathrm{B} i_\mathrm{A} = A_4 i_\mathrm{A}$.

where $A_4 = R_\mathrm{B}$.

Inverting Amplifier Example Problem

Design a system with output $v_o = H(x)$, where process variable x is water level, $x \in [0\,\mathrm{m}, 1\,\mathrm{m}]$, and $H(x) = 10x\frac{\mathrm{V}}{\mathrm{m}}$.

This is the same as the non-inverting amplifier problem.

Solution:

Use same float-and-cable system as in previous example problem.

Use a $100\,\mathrm{k}\Omega$ two-terminal variable resistor:

$H_\mathrm{t}(x) = 100x\frac{\mathrm{k}\Omega}{\mathrm{m}}$.

The variable resistor will be the “input” to the inverting amplifier.

Conditioning-Circuit Function Derivation

So far: $H(x) = 10x\frac{\mathrm{V}}{\mathrm{m}}$ (given) and $H_{\mathrm{t}}(x) = 100x\frac{\mathrm{k\Omega}}{\mathrm{m}}$ (choice of transducer).

Solve for H_{c} in:

$$H_{\mathrm{c}}(H_{\mathrm{t}}(x)) = H(x)$$

Let $y = H_{\mathrm{t}}(x) = 100x\frac{\mathrm{k\Omega}}{\mathrm{m}}$.

Then $x = 0.01y\frac{\mathrm{m}}{\mathrm{k\Omega}}$.

Substituting:

$$\begin{aligned} H_{\mathrm{c}}(y) &= H\left(0.01y\frac{\mathrm{m}}{\mathrm{k\Omega}}\right) \\ &= 10\left(0.01y\frac{\mathrm{m}}{\mathrm{k\Omega}}\right)\frac{\mathrm{V}}{\mathrm{m}} \\ &= 0.1y\frac{\mathrm{V}}{\mathrm{k\Omega}} \end{aligned}$$

For the inverting amplifier used as a resistance-to-voltage converter:

$H_{\mathrm{c}}(R_{\mathrm{B}}) = A_2 R_{\mathrm{B}}$.

$R_{\mathrm{B}} \to y$ and $A_2 \to 0.1\frac{\mathrm{V}}{\mathrm{k\Omega}}$.

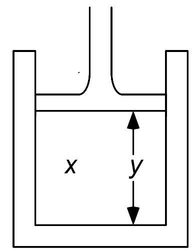
Choose R_{A} and v_{A} so that the following equation is satisfied:

$$0.1\frac{\mathrm{V}}{\mathrm{k\Omega}} = -\frac{v_{\mathrm{A}}}{R_{\mathrm{A}}}$$

For example, $R_{\mathrm{A}} = 60\,\mathrm{k\Omega}$ and $v_{\mathrm{A}} = -6\,\mathrm{V}$.

Another Inverting Amplifier Sample Problem

Design a system with output $v_o = H(x)$, where process variable x is pressure in a sealed cylinder, $x \in [100\,\mathrm{kPa}, 1000\,\mathrm{kPa}]$, and $H(x) = \frac{x}{100\,\mathrm{kPa}}\mathrm{V}$. The cylinder has an area of $100\,\mathrm{cm}^2$. The piston can reach a maximum height of $10\,\mathrm{cm}$ at which point the pressure will be $100\,\mathrm{kPa}$. The cylinder contents is held at a constant temperature.



Plan: Deduce pressure by measuring the position of the piston.

Ideal gas law: $PS = n\Re T$,

where P is the pressure, S is the volume, n is the number of particles, $\Re$ is the *universal gas constant*, and T is the temperature.

Since the cylinder is sealed, n is constant.

Since a constant temperature is maintained, T is constant.

Then: $PS = n\Re T = 100\,\mathrm{kPa}\,10\,\mathrm{cm}\,100\,\mathrm{cm}^2 = 10^5\,\mathrm{kPa}\,\mathrm{cm}^3$.

Solution Plan:

Compute position of piston, y, in terms of pressure, x.

Measure position of piston with variable resistor.

Find conversion circuit to produce $H(x)$.

Transducer(s)

Two transducers are being used:

- Pressure-to-position. (The piston.) Use notation $y = H_{\mathrm{t}1}(x)$.
- Position-to-resistance. Use notation $z = H_{\mathrm{t}2}(y)$.

Pressure to Position

Recall: $PS = 10^5\,\mathrm{kPa}\,\mathrm{cm}^3$.

Here $P \to x$

…and $S \to y100\,\mathrm{cm}^2$.

So: $xy100\,\mathrm{cm}^2 = 10^5\,\mathrm{kPa}\,\mathrm{cm}^3$.

Or: $y = \frac{10^5\,\mathrm{kPa}\,\mathrm{cm}^3}{x100\,\mathrm{cm}^2} = \frac{10^3\,\mathrm{kPa}\,\mathrm{cm}}{x} = H_{\mathrm{t}1}(x)$.

Position to Resistance

Use a $5\,\mathrm{k\Omega}$ variable resistor.

Connect it such that $H_{\mathrm{t}2}(y) = \frac{y}{10\,\mathrm{cm}}5\,\mathrm{k\Omega}$.

Conversion Circuit Function

Desired output: $H(x) = \frac{x}{100\,\mathrm{kPa}}\mathrm{V}$.

$$H_{\mathrm{c}}(H_{\mathrm{t}2}(H_{\mathrm{t}1}(x))) = H(x)$$

Let $z = H_{\mathrm{t}2}(H_{\mathrm{t}1}(x)) = 5\times 10^5\frac{\mathrm{kPa}}{x}\,\Omega$.

Then: $x = 5\times 10^5\frac{\mathrm{kPa}}{z}\,\Omega$.

Substituting:

$$\begin{aligned} H_{\mathrm{c}}(H_{\mathrm{t}2}(H_{\mathrm{t}1}(x))) &= H(x) \\ H_{\mathrm{c}}(z) &= H(5\times 10^5\frac{\mathrm{kPa}}{z}\,\Omega) \\ &= \frac{5}{z}\,\mathrm{k\Omega}\,\mathrm{V} \end{aligned}$$

Conversion Circuit Choice

Use inverting amplifier as inverted-resistance-to-voltage converter.

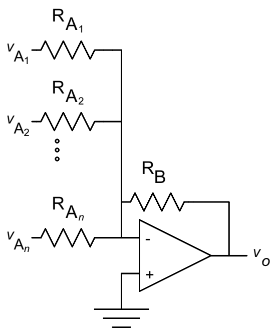
$H_{\mathrm{c}}(R_{\mathrm{A}}) = \frac{A_3}{R_{\mathrm{A}}}$, where $A_3 = -R_{\mathrm{B}}v_{\mathrm{A}}$.

$R_{\mathrm{A}} \to z$ and $A_3 \to 5\,\mathrm{k\Omega}\,\mathrm{V}$.

Choose R_{B} and v_{A} so that $5\,\mathrm{k\Omega}\,\mathrm{V} = -R_{\mathrm{B}}v_{\mathrm{A}}$.

For example, $\boxed{R_{\mathrm{B}} = 50\,\mathrm{k\Omega} \text{ and } v_{\mathrm{A}} = -10\,\mathrm{V}}$.

Summing Amplifier



$$v_o = -R_\mathrm{B} \sum_{i=1}^{n} \frac{v_{\mathrm{A_i}}}{R_{\mathrm{A_i}}}.$$

Applications

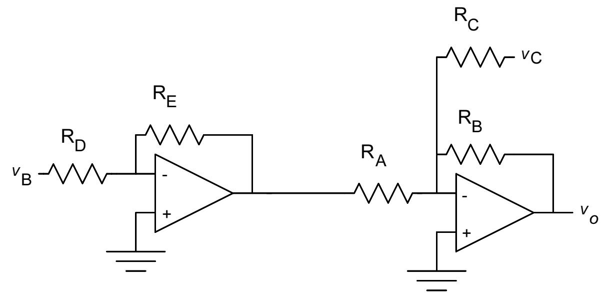
Adding response of several transducers.

Adding "a constant" to the output of a transducer.

Gain/Offset Circuit

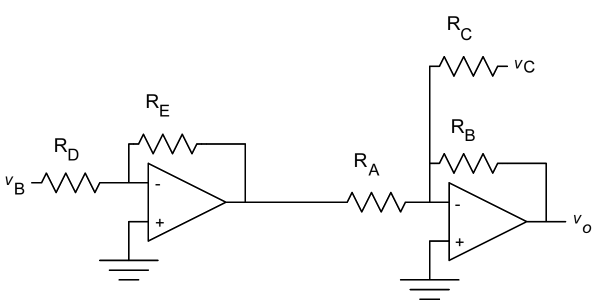
Frequently used conditioning circuit.

Uses one inverting amplifier and one summing amplifier.



$$v_o = \frac{R_\mathrm{B} v_\mathrm{B}}{R_\mathrm{D} R_\mathrm{A}} \left(R_\mathrm{E} - \frac{v_\mathrm{C} R_\mathrm{D} R_\mathrm{A}}{v_\mathrm{B} R_\mathrm{C}} \right).$$

Gain/Offset Circuit



$$v_o = \frac{R_\mathrm{B} v_\mathrm{B}}{R_\mathrm{D} R_\mathrm{A}} \left(R_\mathrm{E} - \frac{v_\mathrm{C} R_\mathrm{D} R_\mathrm{A}}{v_\mathrm{B} R_\mathrm{C}} \right).$$

$v_o = A_5 \left(R_\mathrm{E} - O_5 \right).$

$H_\mathrm{c}(R_\mathrm{E}) = A_5 \left(R_\mathrm{E} - O_5 \right).$

In this form,

- R_E is the input,
- $A_5 = \dfrac{R_\mathrm{B} v_\mathrm{B}}{R_\mathrm{D} R_\mathrm{A}}$ determines the *gain*,
- and $O_5 = \dfrac{v_\mathrm{C} R_\mathrm{D} R_\mathrm{A}}{v_\mathrm{B} R_\mathrm{C}}$ determines the *offset*.

Note that offset can be changed without affecting gain.

Gain/Offset Circuit Example

Design a system with output $v_o = H(x)$, where process variable x is water level, $x \in [0\,\mathrm{m}, 1\,\mathrm{m}]$, and $H(x) = 10x\frac{\mathrm{V}}{\mathrm{m}}$.

This is identical to an earlier problem.

However, it will be solved using a different transducer.

Transducer function will account for the small deviation from perfection.

Transducer:

$H_\mathrm{t}(x) = e_1 \frac{x}{\mathrm{m}} 5\,\mathrm{k}\Omega + e_2,$
where $e_1 = 0.91$ and $e_2 = 37\,\Omega$.

(If $e_1 = 1$ and $e_2 = 0$ then the transducer would be perfect.)

The conditioning circuit should be designed to give the proper output, taking into account e_1 and e_2.

Proceeding in the usual manner:

Let $y = H_t(x) = e_1 \frac{x}{\mathrm{m}} 5\,\mathrm{k}\Omega + e_2$.

Then $x = \frac{\mathrm{m}}{e_1 5\,\mathrm{k}\Omega}(y - e_2)$.

$$\begin{aligned} H_c(H_t(x)) &= H(x) \\ H_c(y) &= 10\frac{\mathrm{V}}{\mathrm{m}} \frac{\mathrm{m}}{e_1 5\,\mathrm{k}\Omega}(y - e_2) \\ &= \frac{2\,\mathrm{V}}{e_1\,\mathrm{k}\Omega}(y - e_2) \end{aligned}$$

Looks like a gain/offset circuit.

$A_5 \to \frac{2\,\mathrm{V}}{e_1\,\mathrm{k}\Omega}$ and $O_5 \to e_2$.

Choose component values so that following are simultaneously satisfied:

$\frac{2\,\mathrm{V}}{e_1\,\mathrm{k}\Omega} = \frac{R_\mathrm{B} v_\mathrm{B}}{R_\mathrm{D} R_\mathrm{A}}$ and $e_2 = \frac{v_\mathrm{C} R_\mathrm{D} R_\mathrm{A}}{v_\mathrm{B} R_\mathrm{C}}$

Choose reasonable values for R_A, R_D, v_B, and v_C.

$v_\mathrm{B} = 5\,\mathrm{V}$ and $R_\mathrm{D} = 5\,\mathrm{k}\Omega$.

Possible reasons: a 5 V supply is available.

Current through transducer (R_E) will be 1 mA, not too large or small for many cases.

$R_\mathrm{A} = 10\,\mathrm{k}\Omega$ and $v_\mathrm{C} = 5\,\mathrm{V}$.

Solving equations then yields:

$R_\mathrm{B} = 22.0\,\mathrm{k}\Omega$.

$R_\mathrm{C} = 1.35\,\mathrm{M}\Omega$.