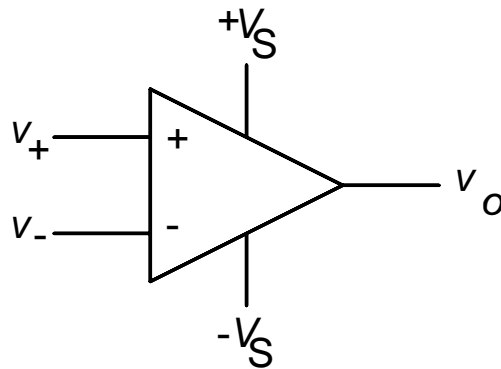


## Operational Amplifiers

These are common components in conditioning circuits.

There are two inputs,  $v_+$  and  $v_-$ , two power supplies,  $+V_s$  and  $-V_s$ , and an output,  $v_o$ .



$v_o = \min\{+V_s, \max\{-V_s, (v_+ - v_-)A\}\}$ , where  $A$  is the op-amp gain.

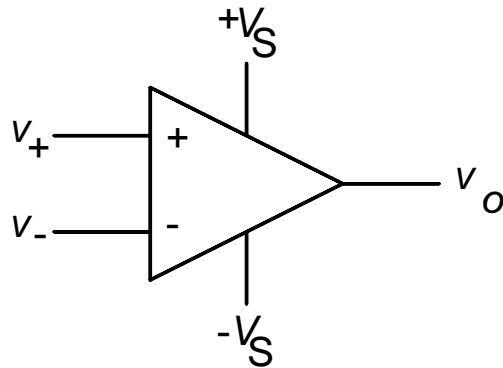
Ignoring saturation,  $v_o = A(v_+ - v_-)$ .

### Ideal Op-Amp Properties

Infinite input impedance.

Infinite gain. ( $A = \infty$ )

Zero output impedance.



### Where to Find Ideal Op-Amps

An electronics textbook.

However, in certain circuits a real op-amp performs almost the same as an ideal op-amp would.

### Simplifying Assumptions

Current into inputs is zero.

When used in a negative feedback configuration,  $v_+ = v_-$ .

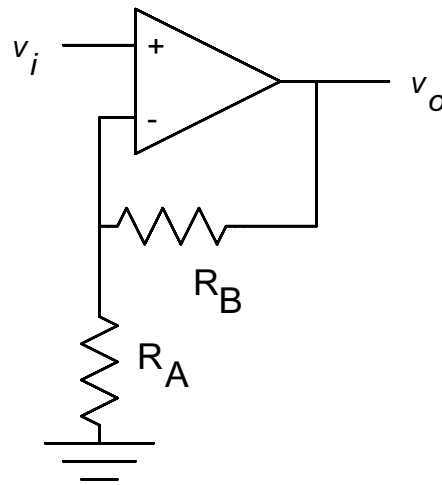
### Op-Amp Circuits to be Covered

Non-inverting amplifier.

Inverting amplifier.

Summing amplifier.

## Non-Inverting Amplifier



$$v_o = \frac{R_A + R_B}{R_A} v_i.$$

## Versatility of Inverting Amplifier

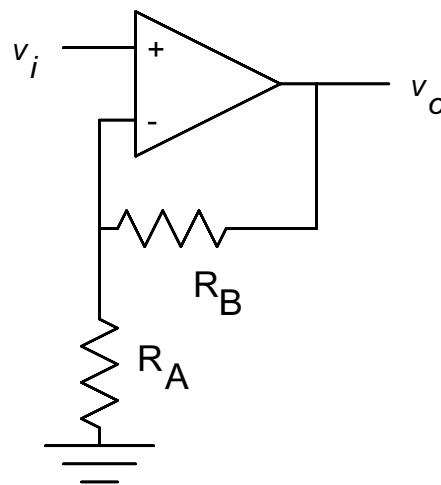
### Use of Non-Inverting Amplifier in Conditioning Circuits

$$v_o = \frac{R_A + R_B}{R_A} v_i.$$

Traditional Use, Voltage Amplifier

Input is  $v_i$ , output is  $v_o$ .

$$H_c(v_i) = \frac{R_A + R_B}{R_A} v_i$$



## Non-Inverting Amplifier Example Problem

*Design a system with output  $v_o = H(x)$ , where process variable  $x$  is water level,  $x \in [0 \text{ m}, 1 \text{ m}]$ , and  $H(x) = 10x \frac{\text{V}}{\text{m}}$ .*

Note: most example problems will not be as complete as the archetypical problem covered earlier.

### Solution:

Use same float-and-cable system as in previous example problem.

Use  $100 \text{ k}\Omega$  three-terminal variable resistor with  $1 \text{ V}$  voltage source across fixed terminals:

$$H_t(x) = 1x \frac{\text{V}}{\text{m}}.$$

Problem will be solved two ways:

First way, we know what kind of conditioning circuit is needed.

Second way, we have to determine algebraically the type of conditioning circuit needed.

### First Way: Use Non-Inverting Amplifier

Obviously, all that is needed is an amplifier with a gain of 10. A non-inverting amplifier will do.

Then:

$$H(x) = H_c(H_t(x)) = A \left( 1x \frac{\text{V}}{\text{m}} \right)$$
$$10x \frac{\text{V}}{\text{m}} = A \left( 1x \frac{\text{V}}{\text{m}} \right)$$
$$A = 10$$

So choose resistors such that  $(R_A + R_B)/R_A = 10$ .

For example,  $R_A = 10 \text{ k}\Omega$  and  $R_B = 90 \text{ k}\Omega$ .

## Second Way: Derive Conditioning-Circuit Function

Pretend we don't know that a simple amplifier is needed.

$$H(x) = H_c(H_t(x))$$

We need to solve for  $H_c$ .

$$\text{Let } y = H_t(x) = 1x \frac{\text{V}}{\text{m}}.$$

$$\text{Then } x = H_t^{-1}(y) = 1y \frac{\text{m}}{\text{V}}.$$

Substituting:

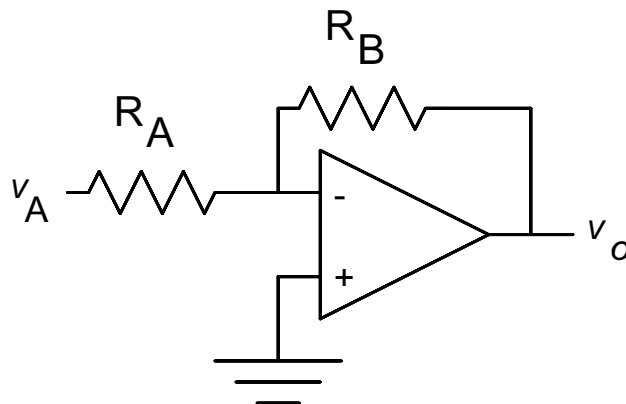
$$\begin{aligned} H\left(1y \frac{\text{m}}{\text{V}}\right) &= H_c(y) \\ H_c(y) &= H\left(1y \frac{\text{m}}{\text{V}}\right) = 10\left(1y \frac{\text{m}}{\text{V}}\right) \\ &= 10y \end{aligned}$$

Therefore our conditioning circuit needs to multiply  $y$ , a voltage, by a constant.

A non-inverting amplifier will do just that.

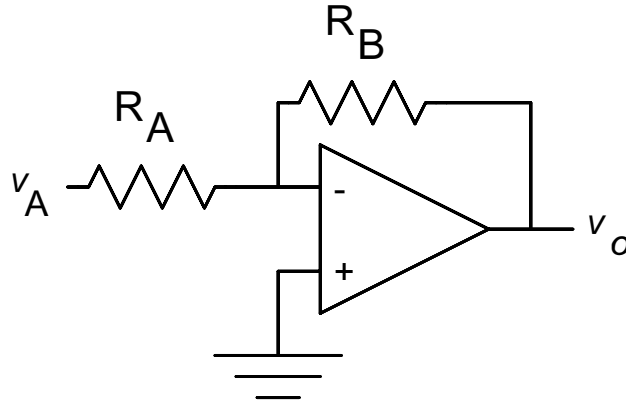
(The remainder of the solution is identical to the first way.)

## Inverting Amplifier



$$v_o = -\frac{R_B}{R_A} v_A.$$





$$\text{Output } v_o = -\frac{R_B}{R_A}v_A.$$

Traditional Use, Voltage Amplifier

Input is  $v_A$  and output is  $v_o$ .

$$H_c(v_A) = -\frac{R_B}{R_A}v_A = A_1v_A$$

where  $A_1 = -\frac{R_B}{R_A}$ .

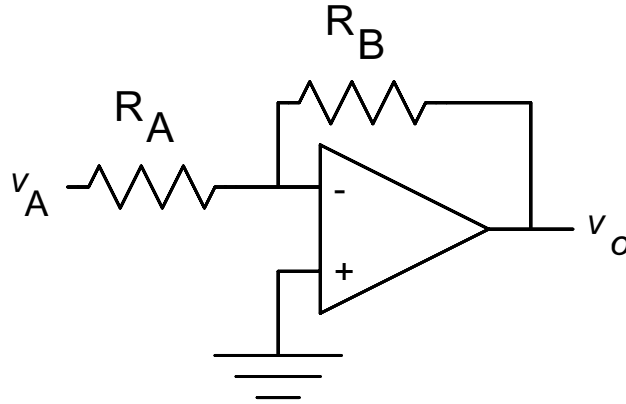
Resistance to Voltage Converter

Input is  $R_B$ , output is  $v_o$ .

$$H_c(R_B) = -\frac{R_B}{R_A}v_A = A_2R_B$$

where  $A_2 = -v_A/R_A$ .

Here,  $v_A$  is a fixed voltage, buried in the constant  $A_2$ .



### Inverted Resistance to Voltage Converter

Input is  $R_A$ , output is  $v_o$ .

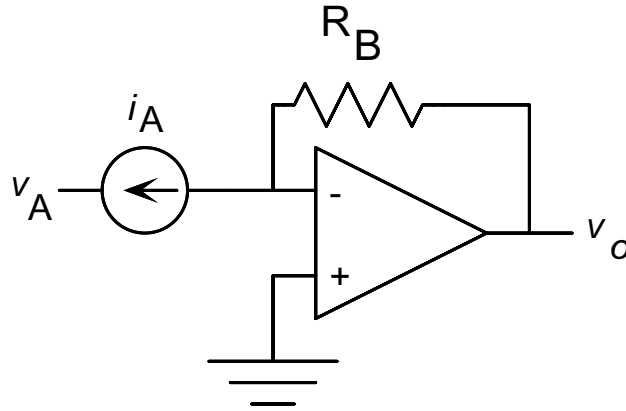
$$H_c(R_A) = -\frac{R_B}{R_A}v_A = A_3/R_A$$

where  $A_3 = -R_B v_A$ .

$v_A$  is a fixed voltage here also.

## Current to Voltage Converter

This circuit is similar to the inverting amplifier.



Input is  $i_A$ , output is  $v_o$ .

$$H_c(i_A) = R_B i_A = A_4 i_A.$$

where  $A_4 = R_B$ .

## Inverting Amplifier Example Problem

*Design a system with output  $v_o = H(x)$ , where process variable  $x$  is water level,  $x \in [0 \text{ m}, 1 \text{ m}]$ , and  $H(x) = 10x \frac{\text{V}}{\text{m}}$ .*

This is the same as the non-inverting amplifier problem.

**Solution:**

Use same float-and-cable system as in previous example problem.

Use a  $100 \text{ k}\Omega$  two-terminal variable resistor:

$$H_t(x) = 100x \frac{\text{k}\Omega}{\text{m}}.$$

The variable resistor will be the “input” to the inverting amplifier.

## Conditioning-Circuit Function Derivation

So far:  $H(x) = 10x \frac{\text{V}}{\text{m}}$  (given) and  $H_t(x) = 100x \frac{\text{k}\Omega}{\text{m}}$  (choice of transducer).

Solve for  $H_c$  in:

$$H_c(H_t(x)) = H(x)$$

Let  $y = H_t(x) = 100x \frac{\text{k}\Omega}{\text{m}}$ .

Then  $x = 0.01y \frac{\text{m}}{\text{k}\Omega}$ .

Substituting:

$$\begin{aligned} H_c(y) &= H\left(0.01y \frac{\text{m}}{\text{k}\Omega}\right) \\ &= 10\left(0.01y \frac{\text{m}}{\text{k}\Omega}\right) \frac{\text{V}}{\text{m}} \\ &= 0.1y \frac{\text{V}}{\text{k}\Omega} \end{aligned}$$

For the inverting amplifier used as a resistance-to-voltage converter:

$$H_c(R_B) = A_2 R_B.$$

$$R_B \rightarrow y \text{ and } A_2 \rightarrow 0.1 \frac{\text{V}}{\text{k}\Omega}.$$

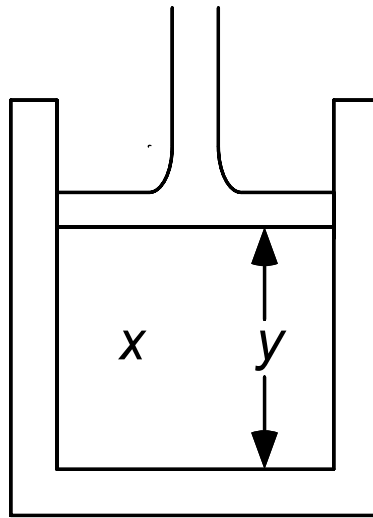
Choose  $R_A$  and  $v_A$  so that the following equation is satisfied:

$$0.1 \frac{\text{V}}{\text{k}\Omega} = -\frac{v_A}{R_A}$$

For example,  $R_A = 60 \text{ k}\Omega$  and  $v_A = -6 \text{ V}$ .

## Another Inverting Amplifier Sample Problem

Design a system with output  $v_o = H(x)$ , where process variable  $x$  is pressure in a sealed cylinder,  $x \in [100 \text{ kPa}, 1000 \text{ kPa}]$ , and  $H(x) = \frac{x}{100 \text{ kPa}} V$ . The cylinder has an area of  $100 \text{ cm}^2$ . The piston can reach a maximum height of  $10 \text{ cm}$  at which point the pressure will be  $100 \text{ kPa}$ . The cylinder contents is held at a constant temperature.



Plan: Deduce pressure by measuring the position of the piston.

Ideal gas law:  $PS = n\mathfrak{R}T$ ,

where  $P$  is the pressure,  $S$  is the volume,  $n$  is the number of particles,  $\mathfrak{R}$  is the *universal gas constant*, and  $T$  is the temperature.

Since the cylinder is sealed,  $n$  is constant.

Since a constant temperature is maintained,  $T$  is constant.

Then:  $PS = n\mathfrak{R}T = 100 \text{ kPa } 10 \text{ cm } 100 \text{ cm}^2 = 10^5 \text{ kPa cm}^3$ .

Solution Plan:

Compute position of piston,  $y$ , in terms of pressure,  $x$ .

Measure position of piston with variable resistor.

Find conversion circuit to produce  $H(x)$ .

Transducer(s)

Two transducers are being used:

- Pressure-to-position. (The piston.) Use notation  $y = H_{t1}(x)$ .
- Position-to-resistance. Use notation  $z = H_{t2}(y)$ .

Pressure to Position

Recall:  $PS = 10^5 \text{ kPa cm}^3$ .

Here  $P \rightarrow x$

...and  $S \rightarrow y100 \text{ cm}^2$ .

So:  $xy100 \text{ cm}^2 = 10^5 \text{ kPa cm}^3$ .

$$\text{Or: } y = \frac{10^5 \text{ kPa cm}^3}{x100 \text{ cm}^2} = \frac{10^3 \text{ kPa cm}}{x} = H_{t1}(x).$$

Position to Resistance

Use a  $5 \text{ k}\Omega$  variable resistor.

Connect it such that  $H_{t2}(y) = \frac{y}{10 \text{ cm}} 5 \text{ k}\Omega$ .

### Conversion Circuit Function

Desired output:  $H(x) = \frac{x}{100 \text{ kPa}} \text{ V}$ .

$$H_c(H_{t2}(H_{t1}(x))) = H(x)$$

Let  $z = H_{t2}(H_{t1}(x)) = 5 \times 10^5 \frac{\text{kPa}}{z} \Omega$ .

Then:  $x = 5 \times 10^5 \frac{\text{kPa}}{z} \Omega$ .

Substituting:

$$H_c(H_{t2}(H_{t1}(x))) = H(x)$$

$$\begin{aligned} H_c(z) &= H\left(5 \times 10^5 \frac{\text{kPa}}{z} \Omega\right) \\ &= \frac{5}{z} \text{ k}\Omega \text{ V} \end{aligned}$$

### Conversion Circuit Choice

Use inverting amplifier as inverted-resistance-to-voltage converter.

$$H_c(R_A) = \frac{A_3}{R_A}, \text{ where } A_3 = -R_B v_A.$$

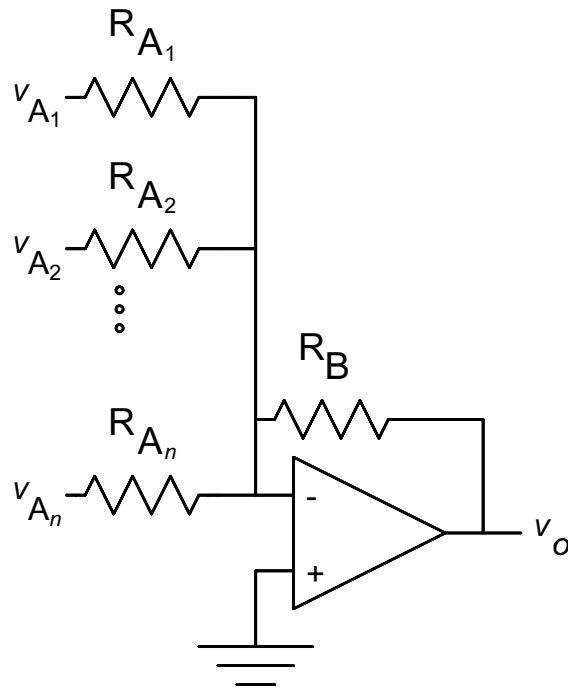
$$R_A \rightarrow z \text{ and } A_3 \rightarrow 5 \text{ k}\Omega \text{ V}.$$

Choose  $R_B$  and  $v_A$  so that  $5 \text{ k}\Omega \text{ V} = -R_B v_A$ .

For example,  $\boxed{R_B = 50 \text{ k}\Omega \text{ and } v_A = -10 \text{ V}}$ .



## Summing Amplifier



$$v_o = -R_B \sum_{i=1}^n \frac{v_{A_i}}{R_{A_i}}.$$

## Applications

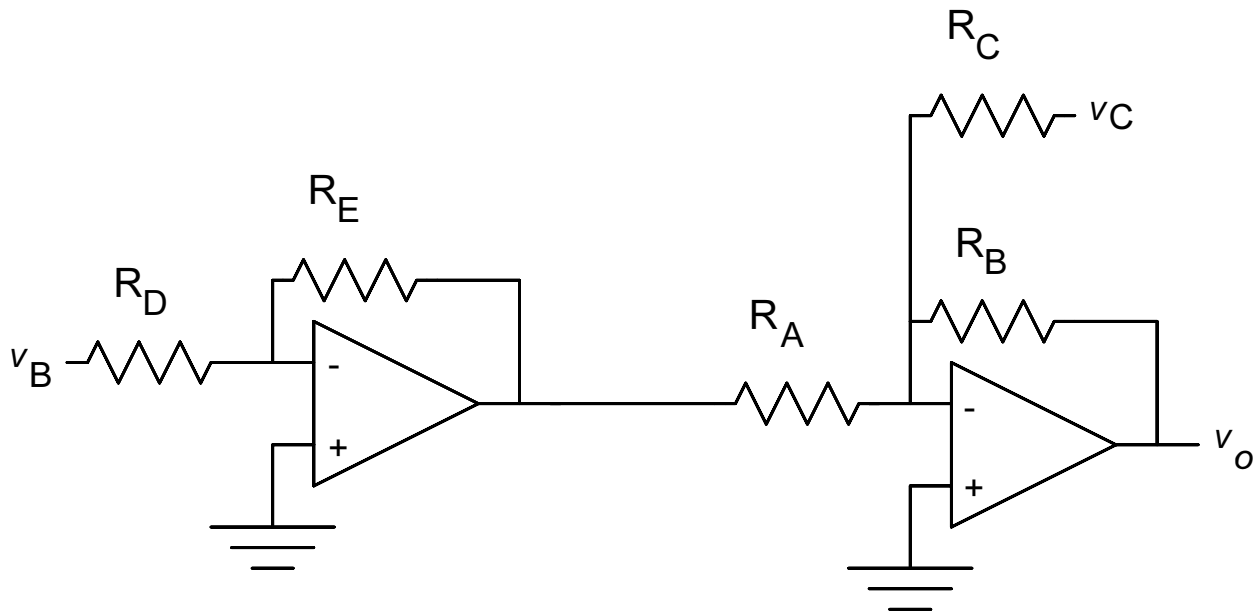
Adding response of several transducers.

Adding “a constant” to the output of a transducer.

## Gain/Offset Circuit

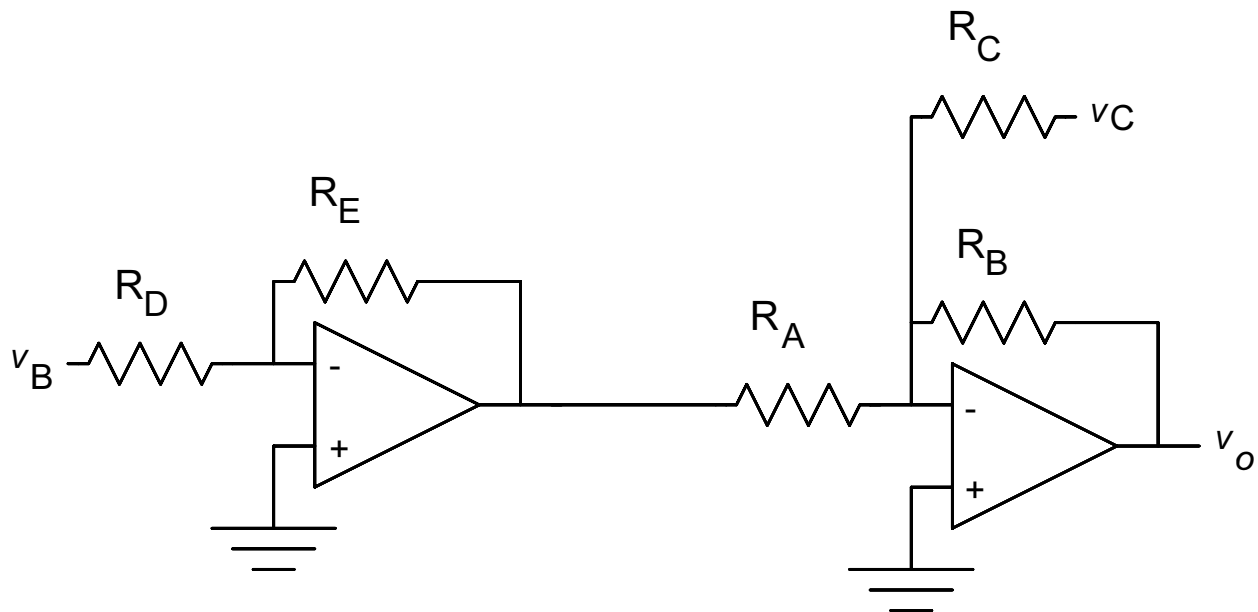
Frequently used conditioning circuit.

Uses one inverting amplifier and one summing amplifier.



$$v_o = \frac{R_B v_B}{R_D R_A} \left( R_E - \frac{v_C R_D R_A}{v_B R_C} \right).$$

## Gain/Offset Circuit



$$v_o = \frac{R_B v_B}{R_D R_A} \left( R_E - \frac{v_C R_D R_A}{v_B R_C} \right).$$

$$v_o = A_5 (R_E - O_5).$$

$$H_c(R_E) = A_5 (R_E - O_5).$$

In this form,

- $R_E$  is the input,
- $A_5 = \frac{R_B v_B}{R_D R_A}$  determines the *gain*,
- and  $O_5 = \frac{v_C R_D R_A}{v_B R_C}$  determines the *offset*.

Note that offset can be changed without affecting gain.

## Gain/Offset Circuit Example

*Design a system with output  $v_o = H(x)$ , where process variable  $x$  is water level,  $x \in [0 \text{ m}, 1 \text{ m}]$ , and  $H(x) = 10x \frac{\text{V}}{\text{m}}$ .*

This is identical to an earlier problem.

However, it will be solved using a different transducer.

Transducer function will account for the small deviation from perfection.

Transducer:

$$H_t(x) = e_1 \frac{x}{\text{m}} 5 \text{ k}\Omega + e_2,$$

where  $e_1 = 0.91$  and  $e_2 = 37 \Omega$ .

(If  $e_1 = 1$  and  $e_2 = 0$  then the transducer would be perfect.)

The conditioning circuit should be designed to give the proper output, taking into account  $e_1$  and  $e_2$ .

Proceeding in the usual manner:

$$\text{Let } y = H_t(x) = e_1 \frac{x}{\text{m}} 5 \text{ k}\Omega + e_2.$$

$$\text{Then } x = \frac{\text{m}}{e_1 5 \text{ k}\Omega} (y - e_2).$$

$$\begin{aligned} H_c(H_t(x)) &= H(x) \\ H_c(y) &= 10 \frac{\text{V}}{\text{m}} \frac{\text{m}}{e_1 5 \text{ k}\Omega} (y - e_2) \\ &= \frac{2 \text{ V}}{e_1 \text{ k}\Omega} (y - e_2) \end{aligned}$$

Looks like a gain/offset circuit.

$$A_5 \rightarrow \frac{2 \text{ V}}{e_1 \text{ k}\Omega} \text{ and } O_5 \rightarrow e_2.$$

Choose component values so that following are simultaneously satisfied:

$$\frac{2 \text{ V}}{e_1 \text{ k}\Omega} = \frac{R_B v_B}{R_D R_A} \text{ and } e_2 = \frac{v_C R_D R_A}{v_B R_C}$$

Choose reasonable values for  $R_A$ ,  $R_D$ ,  $v_B$ , and  $v_C$ .

$$v_B = 5 \text{ V} \text{ and } R_D = 5 \text{ k}\Omega.$$

Possible reasons: a 5 V supply is available.

Current through transducer ( $R_E$ ) will be 1 mA, not too large or small for many cases.

$$R_A = 10 \text{ k}\Omega \text{ and } v_C = 5 \text{ V}.$$

Solving equations then yields:

$$R_B = 22.0 \text{ k}\Omega.$$

$$R_C = 1.35 \text{ M}\Omega.$$