## Comparators

Function: Determine if one voltage is larger than another.
Types of comparators:

- Single-Threshold Op-Amp. Output is a voltage.
- Digital Comparator. Output is a logic level.
- Hysteresis Op-Amp. Output is a voltage, uses two thresholds.


## Applications

- Determine when a voltage exceeds a constant value.
- Determine when one voltage exceeds another.
- Et cetera.

The comparator output might interrupt a computer or perform other functions.

## Single-Threshold Op-Amp Comparator



Here, $v_{+}$and $v_{-}$are inputs.
Transfer function:

$$
H_{\mathrm{c}}\left(v_{+}, v_{-}\right)= \begin{cases}V_{\mathrm{s}}, & \text { if } v_{+}>v_{-} ; \\ -V_{\mathrm{s}}, & \text { if } v_{+} \leq v_{-}\end{cases}
$$

Voltage might be useful for controlling other part of circuit.
But it's not in a convenient form for driving logic circuits.

## Single Threshold Example Problem

Design a system with output $v_{o}=H(x)$ where process variable $x$ is water level, $x \in[0 \mathrm{~m}, 1 \mathrm{~m}]$, and $H(x)=5 \mathrm{~V}$ if $x>30 \mathrm{~cm}$ and $H(x)=0$ otherwise .

Solution:
Use a three-terminal variable resistor to measure water level, as usual.
(Suppose it will later be necessary to measure the exact water level, so that a single switch cannot be used.)

Use an op-amp as a comparator.

## Digital Comparator



Like op-amp comparator except that output is a logical quantity.

$$
H_{\mathrm{c}}\left(v_{+}, v_{-}\right)= \begin{cases}1, & \text { if } v_{+}>v_{-} ; \\ 0, & \text { if } v_{+} \leq v_{-}\end{cases}
$$

This is useful for driving logic circuits.

## Digital Comparator Example Problem

Design a system with output $v=H\left(x_{1}, x_{2}\right)$ where process variables $x_{1}$ and $x_{2}$ are water levels in two different tanks, $x_{1}, x_{2} \in$ [ $0 \mathrm{~m}, 1 \mathrm{~m}$ ], and $H\left(x_{1}, x_{2}\right)=1$ if $\left(x_{1}>30 \mathrm{~cm}\right.$ and $\left.x_{2}>30 \mathrm{~cm}\right)$ or $H\left(x_{1}, x_{2}\right)=0$ otherwise.

Solution:
Use a three-terminal variable resistors to measure water level in each tank, as usual.
(Suppose it will later be necessary to measure the exact water level, so that switches cannot be used.)

Use digital comparators and an and gate.
Note that an op-amp would not be capable of driving logic gates.

# Hysteresis Op-Amp Threshold Detector 

Problem with previous comparators:
Consider a noisy input.

Output of comparator will change many times.
Sometimes this cannot be tolerated.
With hysteresis circuit there are two thresholds, $v_{\text {low }}$ and $v_{\text {high }}$ :

Output will be in state below or above.

- Below-to-above transition occurs at threshold $v_{\text {high }}$.
- Above-to-below transition occurs at threshold $v_{\text {low }}$.

Note that output change is smoother.

Hysteresis Op-Amp Threshold Detector


Circuit Analysis
Given: $v_{\text {low }}$ and $v_{\text {high }}$.
Find: $R_{\mathrm{A}}, R_{\mathrm{B}}, v_{r}$ and $v_{-}$.
Analysis Plan

- Write equation for $v_{\text {in }}=v_{\text {low }}$ in above state.
- Write equation for $v_{\text {in }}=v_{\text {high }}$ in below state.
- Combine and solve for unknowns.

Hysteresis Op-Amp Threshold Detector


Circuit Analysis
Given: $v_{\text {low }}$ and $v_{\text {high }}$.
Find: $R_{\mathrm{A}}, R_{\mathrm{B}}, v_{r}$ and $v_{-}$.

## Observations

In above state: Output is $v_{r}$ and threshold is $v_{\text {low }}$.
In below state: Output is $-v_{r}$ and threshold is $v_{\text {high }}$.
When $v_{\text {in }}=v_{\text {low }}$ in above state, $v_{+}=v_{-}$.
Similarly, when $v_{\text {in }}=v_{\text {high }}$ in below state, $v_{+}=v_{-}$.

$$
\frac{R_{\mathrm{A}}}{R_{\mathrm{B}}}=\frac{v_{\text {high }}-v_{\text {low }}}{2 v_{r}} \quad v_{-}=\frac{v_{\text {high }}+v_{\text {low }}}{2\left(1+\frac{R_{\mathrm{A}}}{R_{\mathrm{B}}}\right)}
$$

## Hysteresis Example

Choose components for a threshold detector so that the output is 5 V in the above state and -5 V in the below state for thresholds $v_{\text {low }}=1.7 \mathrm{~V}$ and $v_{\text {high }}=4.1 \mathrm{~V}$.


Use $v_{r}=5 \mathrm{~V}$.
Using formulas...

$$
\frac{R_{\mathrm{A}}}{R_{\mathrm{B}}}=\frac{v_{\text {high }}-v_{\text {low }}}{2 v_{r}}=0.24
$$

Choose $R_{\mathrm{A}}=240 \Omega$ and $R_{\mathrm{B}}=1000 \Omega$.

$$
v_{-}=\frac{v_{\text {high }}+v_{\text {low }}}{2\left(1+\frac{R_{\mathrm{A}}}{R_{\mathrm{B}}}\right)}=2.34 \mathrm{~V} .
$$

