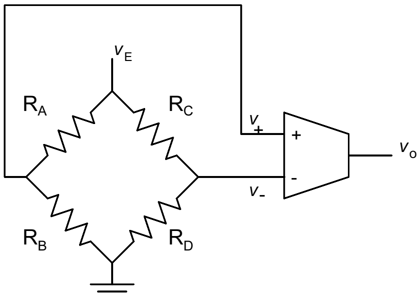
The Wheatstone Bridge



Raison d'être: convert tiny changes in resistance to voltage.

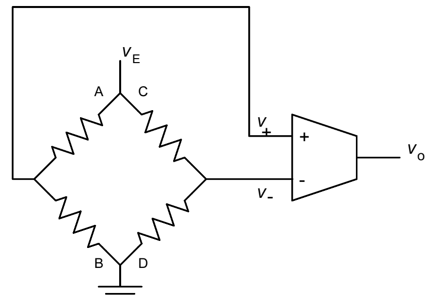
Shown with an *instrumentation amplifier.*
Like an ideal op-amp but with finite gain.

Gain of instrumentation amplifier denoted by A.

$v_o = A(v_+ - v_i)$.

The Wheatstone bridge consists of four *arms.*

$$v_o = A\left(\frac{R_{\rm B}}{R_{\rm A}+R_{\rm B}} - \frac{R_{\rm D}}{R_{\rm C}+R_{\rm D}}\right) v_{\rm E}.$$



Transducer can be placed in one, two, or four arms.

Typical function: $H_{\rm t}(x) = R(1 + xk)$, $xk \ll 1$
where R is the nominal resistance of the transducer and k is a constant.

For simplicity write function as: $H_{\rm t}(x) = R + R_{\rm s}$,
where R is independent of the process-variable value and R_s is dependent on the process-variable value.

Typically, $R \gg R_{\rm s}$.

Usually, need to convert $R_{\rm s}$ to a voltage.

Complementary Pairs

Frequently, transducer pairs can have *complementary responses.*

If so, there are two (usually identical) transducers. . .

. . .positioned so they react *oppositely* to the process variable. . .

. . .so that when their responses are *subtracted.* . .

. . .their response to the process variable *add.* . .

. . .and unwanted quantities *cancel out.*

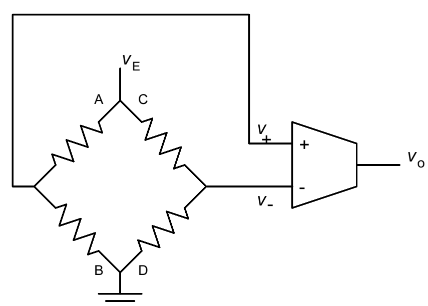
For example, consider:

$H_{\rm t1}(x) = R(1 + xk)$ and $H_{\rm t2}(x) = R(1 - xk)$.

Sum: $H_{\rm t1}(x) + H_{\rm t2}(x) = R$. (Not helpful.)

Difference: $H_{\rm t1}(x) - H_{\rm t2}(x) = 2xk$. (Much better.)

One-Transducer Configuration

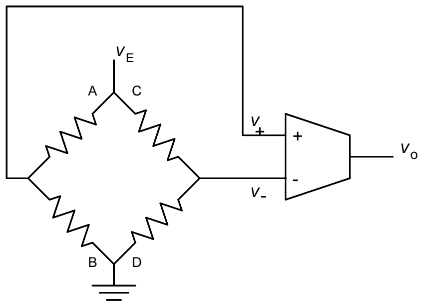


Arm B: $H_{\rm t}(x) = R + R_{\rm s} = R(1 + xk)$.

Other Arms: Resistor of value R.

$$v_o = A\left(\frac{R_{\rm s}}{2(2R + R_{\rm s})}\right) v_{\rm E} \approx A\frac{R_{\rm s}}{4R} v_{\rm E} = A\frac{xk}{4} v_{\rm E}.$$

Two-Transducer Configuration



Arm A: $H_{\mathrm{t}2}(x) = R - R_{\mathrm{s}} = R(1 - xk)$.

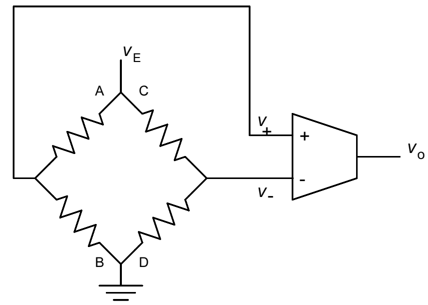
Arm B: $H_{\mathrm{t}1}(x) = R + R_{\mathrm{s}} = R(1 + xk)$.

Other Arms: Resistor of value R.

$$v_o = A\frac{R_{\mathrm{s}}}{2R}v_{\mathrm{E}} = A\frac{xk}{2}v_{\mathrm{E}}.$$

As one might expect, twice as sensitive.

Four-Transducer Configuration



Arms A and D: $H_{\mathrm{t}2}(x) = R - R_{\mathrm{s}} = R(1 - xk)$.

Arms B and C: $H_{\mathrm{t}1}(x) = R + R_{\mathrm{s}} = R(1 + xk)$.

$$v_o = A\frac{R_{\mathrm{s}}}{R}v_{\mathrm{E}} = Axkv_{\mathrm{E}}.$$

Wheatstone Bridge Transfer Functions

Goal

Let $R_{\mathrm{t}} = R \pm R_{\mathrm{s}} = R(1 \pm xk)$ be the transducer response(s).

Assume bridge designed properly.

Need to find two functions:

$H_{\mathrm{c}}(R_{\mathrm{t}}) = \ldots$ and $H_{\mathrm{c}}(R_{\mathrm{s}}) = \ldots$

Both functions are equivalent.

Choose whichever is more convenient.

Four-Transducer Configuration

$$H_{\mathrm{c}}(R_{\mathrm{s}}) = v_o = A\frac{R_{\mathrm{s}}}{R}v_{\mathrm{E}}.$$

Let $R_{\mathrm{t}} = R + R_{\mathrm{s}}$. Then $R_{\mathrm{s}} = R_{\mathrm{t}} - R$.

$$H_{\mathrm{c}}(R_{\mathrm{t}}) = A\frac{R_{\mathrm{t}} - R}{R}v_{\mathrm{E}} = A\left(\frac{R_{\mathrm{t}}}{R} - 1\right)v_{\mathrm{E}}.$$

Two-Transducer Configuration

$$H_{\mathrm{c}}(R_{\mathrm{s}}) = v_o = \frac{A}{2}\frac{R_{\mathrm{s}}}{R}v_{\mathrm{E}}.$$

$$H_{\mathrm{c}}(R_{\mathrm{t}}) = \frac{A}{2}\frac{R_{\mathrm{t}} - R}{R}v_{\mathrm{E}} = \frac{A}{2}\left(\frac{R_{\mathrm{t}}}{R} - 1\right)v_{\mathrm{E}}.$$

One-Transducer Configuration

$$H_{\mathrm{c}}(R_{\mathrm{s}}) = v_o = \frac{A}{4}\frac{R_{\mathrm{s}}}{R}v_{\mathrm{E}}.$$

$$H_{\mathrm{c}}(R_{\mathrm{t}}) = \frac{A}{4}\frac{R_{\mathrm{t}} - R}{R}v_{\mathrm{E}} = \frac{A}{4}\left(\frac{R_{\mathrm{t}}}{R} - 1\right)v_{\mathrm{E}}.$$

Wheatstone Bridge Sample Problem

Design a system with output $v_o = H(x)$, where process variable x is strain and, $x \in [0, 10^{-5}]$, and $H(x) = 10^6 x\,\mathrm{V}$.

Strain will be covered in more detail later.

For now, all we need to know is that strain is dimensionless.

Strain is measured by a *strain gauge*.

Strain gauges frequently used in complementary pairs.

Use strain gauges with response:

$H_{\mathrm{t}}(\epsilon) = R(1 + \epsilon G_f)$,

where ϵ denotes strain and

constant $G_f = 2$.

(G_f called *gauge factor*, a dimensionless quantity.)

Position the two strain gauges to obtain response:

$H_{\mathrm{t}}(x) = R(1 + xG_f)$ and $H_{\mathrm{t}'}(x) = R(1 - xG_f)$.

Derivation of Conditioning Circuit Needed

A Wheatstone bridge is the obvious choice because transducer response is in form $R \pm R_\mathrm{s}$.

Nevertheless, conditioning-circuit derivation will be presented.

$H(x) = H_\mathrm{c}(H_\mathrm{t}(x))$

(Analysis performed as though there were one transducer.)

$y = H_\mathrm{t}(x) = R_\mathrm{t} = R(1 + xG_f)$. Then $x = \dfrac{\frac{y}{R} - 1}{G_f}$.

Then $H_\mathrm{c}(y) = H\left(\dfrac{\frac{y}{R} - 1}{G_f}\right) = 10^6 \dfrac{\frac{y}{R} - 1}{G_f}\,\mathrm{V}$.

Response for two-transducer configuration: $H_\mathrm{c}(R_\mathrm{t}) = \dfrac{A}{2}\left(\dfrac{R_\mathrm{t}}{R} - 1\right)v_\mathrm{E}$.

Choose A and v_E so that $\dfrac{A}{2}v_\mathrm{E} = \dfrac{10^6}{G_f}\,\mathrm{V}$ is satisfied.

For example, $v_\mathrm{E} = 10\,\mathrm{V}$ and $A = 10^5$.