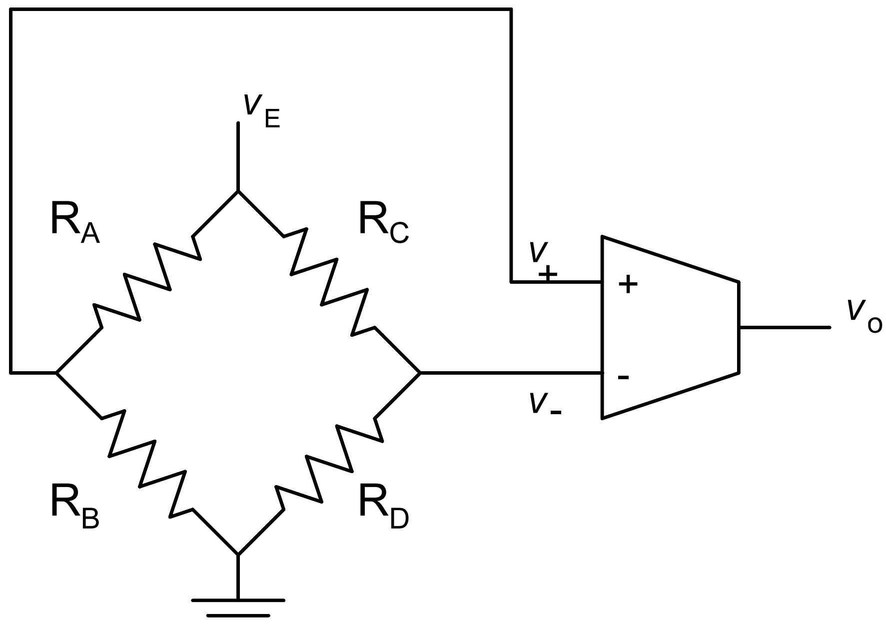
The Wheatstone Bridge



Raison d'être: convert tiny changes in resistance to voltage.

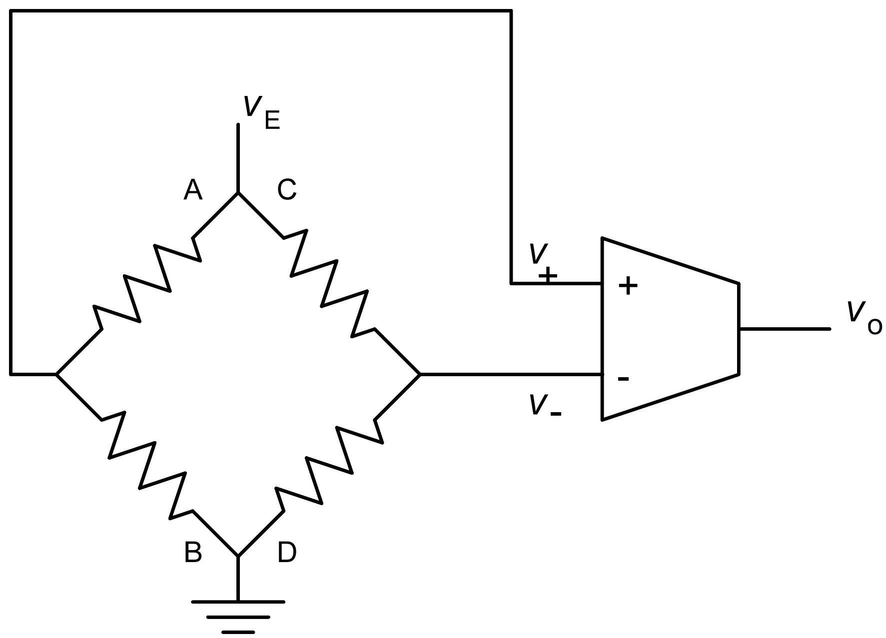
Shown with an *instrumentation amplifier.*
Like an ideal op-amp but with finite gain.

Gain of instrumentation amplifier denoted by A.

$v_o = A(v_+ - v_i)$.

The Wheatstone bridge consists of four *arms*.

$$v_o = A\left(\frac{R_\mathrm{B}}{R_\mathrm{A}+R_\mathrm{B}} - \frac{R_\mathrm{D}}{R_\mathrm{C}+R_\mathrm{D}}\right)v_\mathrm{E}.$$



Transducer can be placed in one, two, or four arms.

Typical function: $H_{\mathrm{t}}(x) = R(1 + xk)$, $xk \ll 1$
where R is the nominal resistance of the transducer and k is a constant.

For simplicity write function as: $H_{\mathrm{t}}(x) = R + R_{\mathrm{s}}$,
where R is independent of the process-variable value and R_s is dependent on the process-variable value.

Typically, $R \gg R_{\mathrm{s}}$.

Usually, need to convert R_{s} to a voltage.

Complementary Pairs

Frequently, transducer pairs can have *complementary responses*.

If so, there are two (usually identical) transducers...

...positioned so they react *oppositely* to the process variable...

...so that when their responses are *subtracted*...

...their response to the process variable *add*...

...and unwanted quantities *cancel out*.

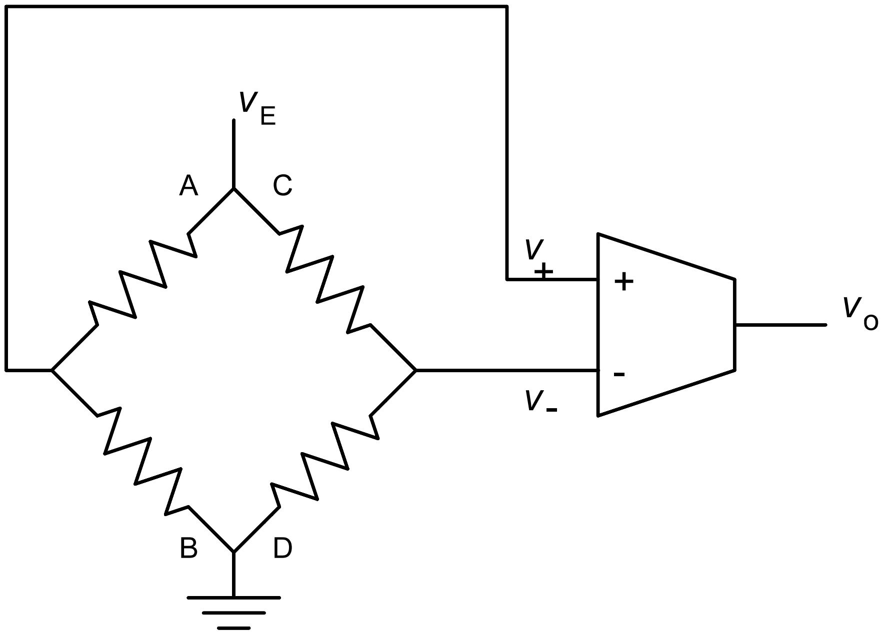
For example, consider:

$$H_{t1}(x) = R(1 + xk) \text{ and } H_{t2}(x) = R(1 - xk).$$

Sum: $H_{t1}(x) + H_{t2}(x) = R$. (Not helpful.)

Difference: $H_{t1}(x) - H_{t2}(x) = 2xk$. (Much better.)

One-Transducer Configuration

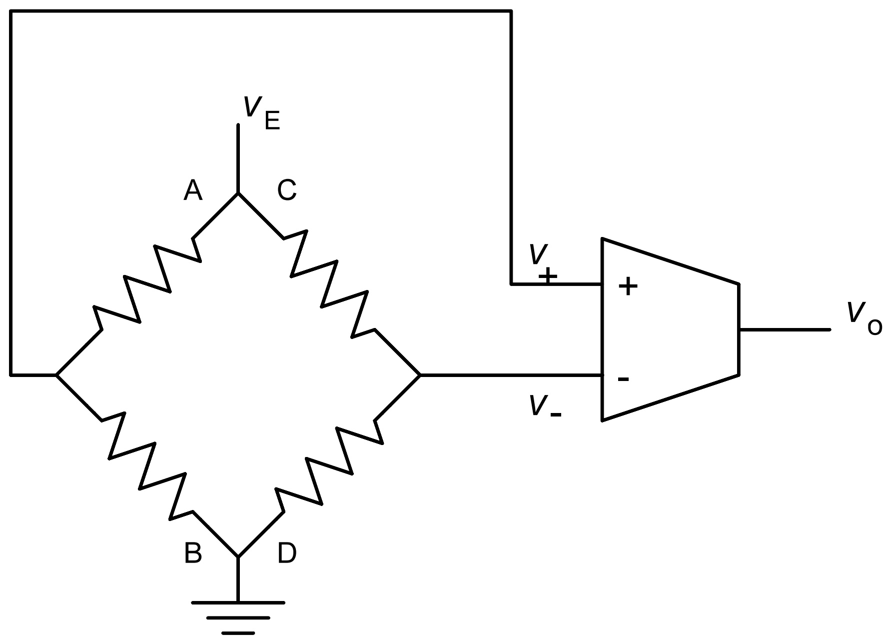


Arm B: $H_{\mathrm{t}}(x) = R + R_{\mathrm{s}} = R(1 + xk)$.

Other Arms: Resistor of value R.

$$v_o = A\left(\frac{R_{\mathrm{s}}}{2(2R + R_{\mathrm{s}})}\right) v_{\mathrm{E}} \approx A\frac{R_{\mathrm{s}}}{4R} v_{\mathrm{E}} = A\frac{xk}{4} v_{\mathrm{E}}.$$

Two-Transducer Configuration



Arm A: $H_{t2}(x) = R - R_s = R(1 - xk)$.

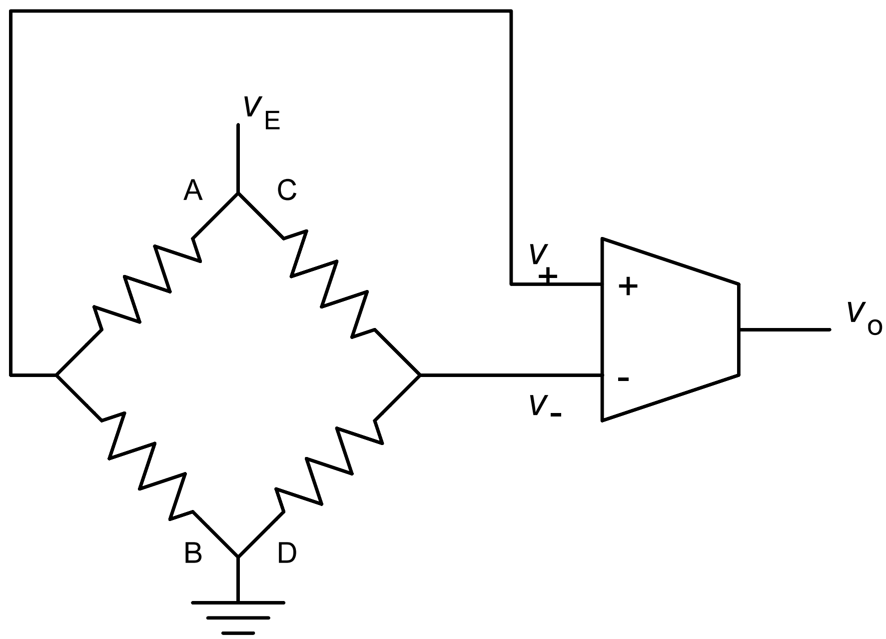
Arm B: $H_{t1}(x) = R + R_s = R(1 + xk)$.

Other Arms: Resistor of value R.

$$v_o = A\frac{R_s}{2R}v_E = A\frac{xk}{2}v_E.$$

As one might expect, twice as sensitive.

Four-Transducer Configuration



Arms A and D: $H_{t2}(x) = R - R_s = R(1 - xk)$.

Arms B and C: $H_{t1}(x) = R + R_s = R(1 + xk)$.

$$v_o = A\frac{R_s}{R}v_E = Axkv_E.$$

Wheatstone Bridge Transfer Functions

Goal

Let $R_{\rm t} = R \pm R_{\rm s} = R(1 \pm xk)$ be the transducer response(s).

Assume bridge designed properly.

Need to find two functions:

$$H_{\rm c}(R_{\rm t}) = \ldots \qquad \text{and } H_{\rm c}(R_{\rm s}) = \ldots$$

Both functions are equivalent.

Choose whichever is more convenient.

Four-Transducer Configuration

$$H_{\rm c}(R_{\rm s}) = v_o = A\frac{R_{\rm s}}{R}v_{\rm E}.$$

Let $R_{\rm t} = R + R_{\rm s}$. Then $R_{\rm s} = R_{\rm t} - R$.

$$H_{\rm c}(R_{\rm t}) = A\frac{R_{\rm t} - R}{R}v_{\rm E} = A\left(\frac{R_{\rm t}}{R} - 1\right)v_{\rm E}.$$

Two-Transducer Configuration

$$H_{\rm c}(R_{\rm s}) = v_o = \frac{A}{2}\frac{R_{\rm s}}{R}v_{\rm E}.$$

$$H_{\rm c}(R_{\rm t}) = \frac{A}{2}\frac{R_{\rm t} - R}{R}v_{\rm E} = \frac{A}{2}\left(\frac{R_{\rm t}}{R} - 1\right)v_{\rm E}.$$

One-Transducer Configuration

$$H_{\rm c}(R_{\rm s}) = v_o = \frac{A}{4}\frac{R_{\rm s}}{R}v_{\rm E}.$$

$$H_{\rm c}(R_{\rm t}) = \frac{A}{4}\frac{R_{\rm t} - R}{R}v_{\rm E} = \frac{A}{4}\left(\frac{R_{\rm t}}{R} - 1\right)v_{\rm E}.$$

Wheatstone Bridge Sample Problem

Design a system with output $v_o = H(x)$, where process variable x is strain and, $x \in [0, 10^{-5}]$, and $H(x) = 10^6 x\,\mathrm{V}$.

Strain will be covered in more detail later.

For now, all we need to know is that strain is dimensionless.

Strain is measured by a *strain gauge*.

Strain gauges frequently used in complementary pairs.

Use strain gauges with response:

$H_{\mathrm{t}}(\epsilon) = R(1 + \epsilon G_f)$,

where ϵ denotes strain and

constant $G_f = 2$.

(G_f called *gauge factor*, a dimensionless quantity.)

Position the two strain gauges to obtain response:

$H_{\mathrm{t}}(x) = R(1 + xG_f)$ and $H_{\mathrm{t}'}(x) = R(1 - xG_f)$.

Derivation of Conditioning Circuit Needed

A Wheatstone bridge is the obvious choice because transducer response is in form $R \pm R_{\mathrm{s}}$.

Nevertheless, conditioning-circuit derivation will be presented.

$H(x) = H_{\mathrm{c}}(H_{\mathrm{t}}(x))$

(Analysis performed as though there were one transducer.)

$y = H_{\mathrm{t}}(x) = R_{\mathrm{t}} = R(1 + xG_f)$. Then $x = \dfrac{\frac{y}{R} - 1}{G_f}$.

Then $H_{\mathrm{c}}(y) = H\left(\dfrac{\frac{y}{R} - 1}{G_f}\right) = 10^6 \dfrac{\frac{y}{R} - 1}{G_f}\,\mathrm{V}$.

Response for two-transducer configuration: $H_{\mathrm{c}}(R_{\mathrm{t}}) = \dfrac{A}{2}\left(\dfrac{R_{\mathrm{t}}}{R} - 1\right)v_{\mathrm{E}}$.

Choose A and v_{E} so that $\dfrac{A}{2} v_{\mathrm{E}} = \dfrac{10^6}{G_f}\,\mathrm{V}$ is satisfied.

For example, $v_{\mathrm{E}} = 10\,\mathrm{V}$ and $A = 10^5$.