Problem 1: Design a system to convert temperature, $x \in[1000 \mathrm{~K}, 1200 \mathrm{~K}]$, to a voltage $H(x)=(x-1000 \mathrm{~K}) \frac{\mathrm{V}}{20 \mathrm{~K}}$. Use a type-B thermocouple and an isothermal block with an integrated temperature sensor. The isothermal block will be exposed to temperatures in $\left[40^{\circ} \mathrm{C}, 60^{\circ} \mathrm{C}\right]$. The response of the thermocouple can be found in NIST Standard Reference Database 60, NIST ITS-90 Thermocouple Database, Version 1.0 and is posted on the web by Omega Engineering at http://www.omega.com/techref/tctables/temper11.html. The integrated temperature sensor has response $H_{\mathrm{its}}(x)=x \frac{\mu \mathrm{~A}}{\mathrm{~K}}$.

The conversion should use only analog circuitry. Design the circuit using linear model(s) of the thermocouple based on the NIST thermocouple tables. Use the integrated temperature sensor to construct an electronic ice bath circuit.

- Show the linear model(s) used in the design. If two linear models are used explain why two were necessary; if one was used explain why only one suffices; if three or more are being considered see the instructor or TA before submitting a solution.
- Show a schematic of the circuit showing all component and supply values.

Overview: The circuit is to generate a voltage linearly related to temperature using a thermocouple, with an output of zero volts at 1000 K . The solution is to use a linear thermocouple model (which is fairly accurate over the temperatures to be encountered) so linear amplifiers can be used. The conditioning circuit needs to perform two functions. First it must "add on" extra voltage to the thermocouple output to account for the connections at the isothermal block. Second, it must apply the necessary gain and offset to the adjusted thermocouple output to get the desired output. These functions are performed by the circuit below.


Both the thermocouple and the ITS are connected to amplifiers which are connected to a summing amplifier. The summing amplifier has an additional constant input, using $R_{C_{C}}$, to generate the necessary offset. Note that the thermocouple voltage is being adjusted at the summing amplifier, after it has passed through an instrumentation amplifier.

Let $H_{\text {TypeB }}(x)$ denote the model function of the Type B thermocouple connected to a $0^{\circ} \mathrm{C}$ isothermal block. Then the voltage of a thermocouple connected to a isothermal block at temperature $T_{R}$ is $H_{\text {TypeB }}(x)-H_{\text {TypeB }}\left(T_{R}\right)$. Call the model function of the integrated temperature sensor $H_{\mathrm{its}}\left(T_{R}\right)$. The output of the circuit is:

$$
\begin{aligned}
v_{o} & =H(x)=(x-1000 \mathrm{~K}) \frac{\mathrm{V}}{20 \mathrm{~K}} \\
& =R_{\mathrm{B}}\left[A\left(H_{\mathrm{TypeB}}(x)-H_{\mathrm{TypeB}}\left(T_{R}\right)\right) \frac{1}{R_{\mathrm{A}}}+R_{\mathrm{i}} H_{\mathrm{its}}\left(T_{R}\right) \frac{1}{R_{A}}-\frac{v_{\mathrm{c}}}{R_{\mathrm{C}}}\right]
\end{aligned}
$$

The response of a Type B thermocouple based on the NIST tables is plotted below. Three temperature ranges are shown, a wide one, one matching the isothermal block temperatures, and one matching the measurement temperatures. In the isothermal block range, $\left[40^{\circ} \mathrm{C}, 60^{\circ} \mathrm{C}\right]$, the change in voltage is small; because the data is to a precision of 0.001 mV the graph has a stair-step appearance.




Two linear models will be used for $H_{\text {TypeB, }}$ one for each temperature range. The linear model for the low temperature range, $\left[40^{\circ} \mathrm{C}, 60^{\circ} \mathrm{C}\right]$, is the equation of a line passing through the end points of the temperature range. At $40^{\circ} \mathrm{C}$ the thermocouple voltage is 0 mV (that is, $H_{\text {TypeB }}\left(40^{\circ} \mathrm{C}\right)=0 \mathrm{mV}$ using the thermocouple tables); at $60^{\circ} \mathrm{C}$ the voltage is 0.006 mV . The model function is

$$
H_{\mathrm{t}-\text { low }}\left(T_{R}\right)=0 \mathrm{mV}+\frac{T_{R}-40^{\circ} \mathrm{C}}{20^{\circ} \mathrm{C}}(0.006 \mathrm{mV}-0 \mathrm{mV})=0.0003\left(x-40^{\circ} \mathrm{C}\right) \frac{\mathrm{mV}}{{ }^{\circ} \mathrm{C}} .
$$

At the endpoints of the high temperature range $H_{\text {TypeB }}\left(726^{\circ} \mathrm{C}\right)=2.611 \mathrm{mV}$ and $H_{\text {TypeB }}\left(926^{\circ} \mathrm{C}\right)=$ 4.178 mV . (Note the conversion to Celsius and that temperatures are truncated to integers.) The model is

$$
\begin{aligned}
H_{\mathrm{t}-\mathrm{high}}(x) & =2.611 \mathrm{mV}+\frac{x-726^{\circ} \mathrm{C}}{200^{\circ} \mathrm{C}}(4.178 \mathrm{mV}-2.611 \mathrm{mV}) \\
& =2.611 \mathrm{mV}+0.007835\left(x-726^{\circ} \mathrm{C}\right) \frac{\mathrm{mV}}{{ }^{\circ} \mathrm{C}} .
\end{aligned}
$$

Re-writing the equation describing the circuit:

$$
\begin{align*}
H(x) & =v_{o}= \\
(x-1000 \mathrm{~K}) \frac{\mathrm{V}}{20 \mathrm{~K}} & =R_{\mathrm{B}}\left[A\left(H_{\mathrm{t}-\mathrm{high}}(x)-H_{\mathrm{t}-\mathrm{low}}\left(T_{R}\right)\right) \frac{1}{R_{\mathrm{A}}}+R_{\mathrm{i}} H_{\mathrm{its}}\left(T_{R}\right) \frac{1}{R_{A}}-\frac{v_{\mathrm{c}}}{R_{\mathrm{C}}}\right] \tag{1}
\end{align*}
$$

Component and supply values could be found by blindly solving this equation using algebraic means. However it is easier and more insightful to solve it in parts, considering their functions.

Summing Amplifier Gain
The summing amplifier combines the thermocouple and ITS outputs and a constant offset and can also provide additional gain. Since the thermocouple and ITS already have amplifiers the summing amplifier gain will be set to a relatively small value of 10 , by choosing $R_{\mathrm{B}}=10 R_{\mathrm{A}}$. A good value of $R_{\mathrm{A}}$ is $1 \mathrm{k} \Omega$.

Instrumentation Amplifier Gain
The instrumentation amplifier gain is chosen so the output has the right "slope." That is, for each change in temperature of one Kelvin the output voltage changes by $\frac{1}{20}$ volts. One could take the derivative of (1) with respect to $x$ and solve for $A$, but it's easier to multiply it out and remove terms that do not contain $x$. That gain can be found by solving for $A$ in

$$
x \frac{\mathrm{~V}}{20 \mathrm{~K}}=R_{\mathrm{B}}\left[A\left(H_{\mathrm{t}-\mathrm{high}^{\prime}}(x)\right) \frac{1}{R_{\mathrm{A}}}\right],
$$

where $H_{\mathrm{t}-\mathrm{high}^{\prime}}(x)=0.007835 x \frac{\mathrm{mV}}{{ }^{\circ} \mathrm{C}}$. Solving, $A=638.2$.
Integrated Temperature Sensor Gain
The ITS gain (using $R_{\mathrm{i}}$ ) and a constant current, $i_{\mathrm{C} 1}$, is chosen to eliminate $H_{\mathrm{t}-\mathrm{low}}\left(T_{R}\right)$ from equation (1) by solving:

$$
-A H_{\mathrm{t}-\mathrm{low}}\left(T_{R}\right) \frac{1}{R_{\mathrm{A}}}+R_{\mathrm{i}} H_{\mathrm{its}}\left(T_{R}\right) \frac{1}{R_{A}}-i_{\mathrm{C} 1}=0
$$

Applying the model functions and solving yields $R_{\mathrm{i}}=191 \Omega$ and $i_{\mathrm{C} 1}=52.14 \mu \mathrm{~A}$. (The current, along with another, will be provided by $v_{\mathrm{c}} / R_{\mathrm{C}}$.)

Offset Voltage
The solution is completed by choosing $R_{\mathrm{C}}$ to satisfy.

$$
(x-1000 \mathrm{~K}) \frac{\mathrm{V}}{20 \mathrm{~K}}=R_{\mathrm{B}}\left[A\left(H_{\mathrm{TypeB}}(x)-H_{\mathrm{TypeB}}\left(T_{R}\right)\right) \frac{1}{R_{\mathrm{A}}}+R_{\mathrm{i}} H_{\mathrm{its}}\left(T_{R}\right) \frac{1}{R_{A}}-\frac{v_{\mathrm{c}}}{R_{\mathrm{C}}}\right] .
$$

That value should provide the current for the ITS, $i_{\mathrm{C} 1}$, and a current to cancel the current from the thermocouple into the summing amplifier at the minimum temperature. That second current is

$$
A H_{\mathrm{t}-\mathrm{high}}\left(726.85^{\circ} \mathrm{C}\right) / R_{\mathrm{A}}=1.666 \mathrm{~mA} .
$$

The two currents flow in the same direction, the sum is 1.719 mA flowing into the summing amplifier. This can be provided by choosing $v_{\mathrm{c}}=10 \mathrm{~V}$ and $R_{\mathrm{C}}=5.819 \mathrm{k} \Omega$.

The circuit could be simplified by connecting the ITS directly to the summing amplifier.

