## Error

*Error* is the difference between an ideal (or correct) value and an actual value.

- Several different *types* of error can be measured.
- An error type can be expressed in several ways.





## Expression of Error

Notation

$\mathcal{I}$ denotes an *ideal value.*

$\mathcal{A}$ denotes an *actual value.*

*Absolute error* defined $|\mathcal{I} - \mathcal{A}|$.

*Percent error* defined $100\dfrac{|\mathcal{I} - \mathcal{A}|}{\mathcal{I}}$ for $\mathcal{I} \neq 0$.

Consider a transducer designed to measure process variables in the range $\mathcal{I} \in [x_{\min}, x_{\max}]$.

*Percent-full-scale error* defined $100\dfrac{|\mathcal{I} - \mathcal{A}|}{x_{\max}}$ for $x_{\max} \neq 0$.





## Types of Error

- *Model Error.*
  Error in transducer model, $H_{\mathrm{t}}$.
- *Repeatability Error.*
  Transducer change from occasion to occasion.
- *Stability Error.*
  Transducer change during use.
- *Calibration Error.*
  Difference between two transducers of same kind.





## Model Error

Let $y = H_{\mathrm{t}}(x)$ denote a transducer output, response, and process variable.

The accuracy of $H_{\mathrm{t}}(x)$ depends upon how well the transducer is understood and how complex a transfer function can be tolerated.

For example, the following are all for the same transducer:

Okay: $H_{\mathrm{t}1}(x) = R_o(1 + ax)$.

Good: $H_{\mathrm{t}2}(x) = R_o(1 + ax + bx^2)$.

Better: $H_{\mathrm{t}3}(x) = R_o(1 + ax + bx^2 + cx^3)$.

Best: $H_{\mathrm{t}4}(0\,^\circ\mathrm{C}) = 100\,\Omega$, $H_{\mathrm{t}4}(0.01\,^\circ\mathrm{C}) = 100.15\,\Omega, \ldots$. (Called a *lookup table.*)

Model error quantifies the accuracy of the transfer function.

Definition of Model Error Quantities

Test conditions: a single measurement. Let $H_{\mathrm{t}}(x)$ denote the transducer response, $x$ denote the process-variable value, and $y$ the quantity measured at at the transducer outputs.

Then: Ideal: $\mathcal{I} = x$, Actual: $\mathcal{A} = H_{\mathrm{t}}^{-1}(y)$.

## Model Error Example

*What is the absolute model error of a transducer having response $H_t(x) = (10x^2 - 5)\,\mathrm{V}$ under test conditions, with process variable $x = 2.130$ and measured transducer output $y = 34.90\,\mathrm{V}$.*

The ideal quantity is $\mathcal{I} = 2.130$.

$H_t^{-1}(y) = \sqrt{\frac{1}{10}\left(\frac{y}{\mathrm{V}} + 5\right)}$.

Based on the transducer $\mathcal{A} = H_t^{-1}(34.9\,\mathrm{V}) = 1.998$.

The absolute error is then, 0.1325.





## Repeatability

Measures how well a transducer performs over time.

**Definition of Repeatability Error Quantities**

Test conditions:

Let $H_t(x)$ denote the transducer response.

Let $x(t)$ denote the value of the process variable at time $t$.

Two measurements are made, at times $t_1$ and $t_2$, $t_1 < t_2$.

The test is set up so that $x(t_1) = x(t_2) = x$ and $x(t_{1.5}) \neq x$ for some $t_1 < t_{1.5} < t_2$.

Let $y_1$ and $y_2$ denote the quantities read at the transducer outputs at times $t_1$ and $t_2$.

Then: Ideal: $\mathcal{I} = H_t^{-1}(y_1)$. Actual: $\mathcal{A} = H_t^{-1}(y_2)$.





## Stability

Measures how well the a transducer measures a steady quantity.

**Definition of Stability Error Quantities**

Test conditions:

Let $H_t(x)$ denote the transducer response.

Let $x(t)$ denote the value of the process variable at time $t$.

Two measurements are made, at times $t_1$ and $t_2$, $t_1 < t_2$.

The test is set up so that $x(t_1) = x(t_2) = x(t_{1.5}) = x$ for all $t_1 < t_{1.5} < t_2$.

Let $y_1$ and $y_2$ denote the quantities read at the transducer outputs at $t_1$ and $t_2$.

Then: Ideal: $\mathcal{I} = H_t^{-1}(y_1)$. Actual: $\mathcal{A} = H_t^{-1}(y_2)$.





## Calibration

Measures how well two transducers of the same type compare.

**Definition of Calibration Error Quantities**

Test conditions:

Let $H_t(x)$ denote the transducer response and $x$ denote the value of the process variable.

A measurement is made with each transducer.

Let $y_1$ and $y_2$ be the quantities read at the transducers' outputs.

Then: Ideal: $\mathcal{I} = H_t^{-1}(y_1)$. Actual: $\mathcal{A} = H_t^{-1}(y_2)$.

## Example

*A type of* integrated temperature sensor *has a response of* $H_{\mathrm{t}}(x) = 7x\frac{\mu\mathrm{A}}{\mathrm{K}}$. *Tests were performed on two such sensors by exposing the sensors to a known temperature, $x$, and measuring their response, $y$, as follows:*

*At time $t_1$ sensor A exposed to $x = 295\,\mathrm{K}$; output $y = 2050\,\mu\mathrm{A}$.*

*At time $t_2$ sensor A exposed to $x = 300\,\mathrm{K}$; output $y = 2085\,\mu\mathrm{A}$.*

*At time $t_3$ sensor A exposed to $x = 295\,\mathrm{K}$; output $y = 2052\,\mu\mathrm{A}$.*

*At time $t_4$ sensor A exposed to $x = 295\,\mathrm{K}$; output $y = 2053\,\mu\mathrm{A}$.*

*At time $t_5$ sensor B exposed to $x = 295\,\mathrm{K}$; output $y = 2040\,\mu\mathrm{A}$.*

*Temperature is held constant from $t_3$ to $t_5$. Find model error, repeatability error, stability error, and calibration error.*





### Inverted Model Function

$x = H_{\mathrm{t}}^{-1}(y) = y\frac{\mathrm{K}}{7\,\mu\mathrm{A}}$.

### Model Error

Use measurement at $t_1$.

$\mathcal{I} = 295.0\,\mathrm{K}$ and $\mathcal{A} = H_{\mathrm{t}}^{-1}(2050\,\mu\mathrm{A}) = 292.9\,\mathrm{K}$.

Percent model error: $\frac{|295.0\,\mathrm{K} - 292.9\,\mathrm{K}|}{295.0\,\mathrm{K}} = 0.71\%$.

Could have used any time to compute model error.





### Repeatability Error

Use measurements at $t_1$ and $t_3$ (since temperature different at $t_2$).

$\mathcal{I} = H_{\mathrm{t}}^{-1}(y(t_1)) = H_{\mathrm{t}}^{-1}(2050\,\mu\mathrm{A}) = 292.9\,\mathrm{K}$.

$\mathcal{A} = H_{\mathrm{t}}^{-1}(y(t_3)) = H_{\mathrm{t}}^{-1}(2052\,\mu\mathrm{A}) = 293.1\,\mathrm{K}$.

Percent repeatability error: $\frac{|292.9\,\mathrm{K} - 293.1\,\mathrm{K}|}{292.9\,\mathrm{K}} = 0.06828\%$.

Note, actual and ideal quantities could be reversed in this example.

Also possible to use $t_1$ and $t_4$.





### Stability Error

Use measurements at $t_3$ and $t_4$ (since temperature held constant in this time range).

$\mathcal{I} = H_{\mathrm{t}}^{-1}(y(t_3)) = H_{\mathrm{t}}^{-1}(2052\,\mu\mathrm{A}) = 293.1\,\mathrm{K}$.

$\mathcal{A} = H_{\mathrm{t}}^{-1}(y(t_4)) = H_{\mathrm{t}}^{-1}(2053\,\mu\mathrm{A}) = 293.3\,\mathrm{K}$.

Percent stability error: $\frac{|293.1\,\mathrm{K} - 293.3\,\mathrm{K}|}{293.1\,\mathrm{K}} = 0.06824\%$.

## Calibration Error

Use measurements at $t_4$ and $t_5$.

$\mathcal{I} = H_t^{-1}(y(t_4)) = H_t^{-1}(2053\,\mu\mathrm{A}) = 293.3\,\mathrm{K}$.

$\mathcal{A} = H_t^{-1}(y(t_5)) = H_t^{-1}(2040\,\mu\mathrm{A}) = 291.4\,\mathrm{K}$.

Percent calibration error: $\dfrac{|293.3\,\mathrm{K} - 291.4\,\mathrm{K}|}{293.3\,\mathrm{K}} = 0.6478\%$.





## Miscellany

Typically, error specially defined for each type of transducer.

The definition includes the exact test circuit and test conditions.

Error measures can be applied to conditioning circuits and anything else that transforms a process variable value.