Error

06-1

Error is the difference between an ideal (or correct) value and an actual value.

- Several different *types* of error can be measured.
- An error type can be expressed in several ways.

Expression of Error

06-2

Notation

$\mathcal{I}$ denotes an *ideal value.*

$\mathcal{A}$ denotes an *actual value.*

Absolute error defined $|\mathcal{I} - \mathcal{A}|$.

Percent error defined $100\dfrac{|\mathcal{I} - \mathcal{A}|}{\mathcal{I}}$ for $\mathcal{I} \neq 0$.

Consider a transducer designed to measure process variables in the range $\mathcal{I} \in [x_{\min}, x_{\max}]$.

Percent-full-scale error defined $100\dfrac{|\mathcal{I} - \mathcal{A}|}{x_{\max}}$ for $x_{\max} \neq 0$.

06-2

Types of Error

06-3

- *Model Error.*
  Error in transducer model, H_{t}.
- *Repeatability Error.*
  Transducer change from occasion to occasion.
- *Stability Error.*
  Transducer change during use.
- *Calibration Error.*
  Difference between two transducers of same kind.

Model Error

Let $y = H_t(x)$ denote a transducer output, response, and process variable.

The accuracy of $H_t(x)$ depends upon how well the transducer is understood and how complex a transfer function can be tolerated.

For example, the following are all for the same transducer:

Okay: $H_{t1}(x) = R_o(1 + ax)$.

Good: $H_{t2}(x) = R_o(1 + ax + bx^2)$.

Better: $H_{t3}(x) = R_o(1 + ax + bx^2 + cx^3)$.

Best: $H_{t4}(0\,^\circ\mathrm{C}) = 100\,\Omega$, $H_{t4}(0.01\,^\circ\mathrm{C}) = 100.15\,\Omega$, ... (Called a *lookup table.*)

Model error quantifies the accuracy of the transfer function.

> Definition of Model Error Quantities
>
> Test conditions: a single measurement. Let $H_t(x)$ denote the transducer response, x denote the process-variable value, and y the quantity measured at at the transducer outputs.
>
> Then: Ideal: $\mathcal{I} = x$, Actual: $\mathcal{A} = H_t^{-1}(y)$.

Model Error Example

06-5

What is the absolute model error of a transducer having response $H_{\mathrm{t}}(x) = (10x^2 - 5)\,\mathrm{V}$ *under test conditions, with process variable* $x = 2.130$ *and measured transducer output* $y = 34.90\,\mathrm{V}$.

The ideal quantity is $\mathcal{I} = 2.130$.

$$H_{\mathrm{t}}^{-1}(y) = \sqrt{\frac{1}{10}\left(\frac{y}{\mathrm{V}} + 5\right)}.$$

Based on the transducer $\mathcal{A} = H_{\mathrm{t}}^{-1}(34.9\,\mathrm{V}) = 1.998$.

The absolute error is then, 0.1325.

Repeatability

06-6

Measures how well a transducer performs over time.

Definition of Repeatability Error Quantities

Test conditions:

Let $H_{\mathrm{t}}(x)$ denote the transducer response.

Let $x(t)$ denote the value of the process variable at time t.

Two measurements are made, at times t_1 and t_2, $t_1 < t_2$.

The test is set up so that $x(t_1) = x(t_2) = x$ and $x(t_{1.5}) \neq x$ for some $t_1 < t_{1.5} < t_2$.

Let y_1 and y_2 denote the quantities read at the transducer outputs at times t_1 and t_2.

Then: Ideal: $\mathcal{I} = H_{\mathrm{t}}^{-1}(y_1)$. Actual: $\mathcal{A} = H_{\mathrm{t}}^{-1}(y_2)$.

06-6

Stability

Measures how well the a transducer measures a steady quantity.

Definition of Stability Error Quantities

Test conditions:

Let $H_{\mathrm{t}}(x)$ denote the transducer response.

Let $x(t)$ denote the value of the process variable at time t.

Two measurements are made, at times t_1 and t_2, $t_1 < t_2$.

The test is set up so that $x(t_1) = x(t_2) = x(t_{1.5}) = x$ for all $t_1 < t_{1.5} < t_2$.

Let y_1 and y_2 denote the quantities read at the transducer outputs at t_1 and t_2.

Then: Ideal: $\mathcal{I} = H_{\mathrm{t}}^{-1}(y_1)$. Actual: $\mathcal{A} = H_{\mathrm{t}}^{-1}(y_2)$.

Calibration

06-8

Measures how well two transducers of the same type compare.

Definition of Calibration Error Quantities

Test conditions:

Let $H_{\mathrm{t}}(x)$ denote the transducer response and x denote the value of the process variable.

A measurement is made with each transducer.

Let y_1 and y_2 be the quantities read at the transducers' outputs.

Then: Ideal: $\mathcal{I} = H_{\mathrm{t}}^{-1}(y_1)$. Actual: $\mathcal{A} = H_{\mathrm{t}}^{-1}(y_2)$.

Example

A type of integrated temperature sensor *has a response of* $H_t(x) = 7x\frac{\mu A}{K}$. *Tests were performed on two such sensors by exposing the sensors to a known temperature,* x*, and measuring their response,* y*, as follows:*

At time t_1 *sensor A exposed to* $x = 295\,K$*; output* $y = 2050\,\mu A$.

At time t_2 *sensor A exposed to* $x = 300\,K$*; output* $y = 2085\,\mu A$.

At time t_3 *sensor A exposed to* $x = 295\,K$*; output* $y = 2052\,\mu A$.

At time t_4 *sensor A exposed to* $x = 295\,K$*; output* $y = 2053\,\mu A$.

At time t_5 *sensor B exposed to* $x = 295\,K$*; output* $y = 2040\,\mu A$.

Temperature is held constant from t_3 *to* t_5*. Find model error, repeatability error, stability error, and calibration error.*

Inverted Model Function

$$x = H_{\mathrm{t}}^{-1}(y) = y\frac{\mathrm{K}}{7\,\mu\mathrm{A}}.$$

Model Error

Use measurement at t_1.

$\mathcal{I} = 295.0\,\mathrm{K}$ and $\mathcal{A} = H_{\mathrm{t}}^{-1}(2050\,\mu\mathrm{A}) = 292.9\,\mathrm{K}$.

Percent model error: $\dfrac{|295.0\,\mathrm{K} - 292.9\,\mathrm{K}|}{295.0\,\mathrm{K}} = 0.71\%$.

Could have used any time to compute model error.

Repeatability Error

Use measurements at t_1 and t_3 (since temperature different at t_2).

$\mathcal{I} = H_{\mathrm{t}}^{-1}(y(t_1)) = H_{\mathrm{t}}^{-1}(2050\,\mu\mathrm{A}) = 292.9\,\mathrm{K}$.

$\mathcal{A} = H_{\mathrm{t}}^{-1}(y(t_3)) = H_{\mathrm{t}}^{-1}(2052\,\mu\mathrm{A}) = 293.1\,\mathrm{K}$.

Percent repeatability error: $\dfrac{|292.9\,\mathrm{K} - 293.1\,\mathrm{K}|}{292.9\,\mathrm{K}} = 0.06828\%$.

Note, actual and ideal quantities could be reversed in this example.

Also possible to use t_1 and t_4.

Stability Error

Use measurements at t_3 and t_4 (since temperature held constant in this time range).

$\mathcal{I} = H_{\rm t}^{-1}(y(t_3)) = H_{\rm t}^{-1}(2052\,\mu\text{A}) = 293.1\,\text{K}.$

$\mathcal{A} = H_{\rm t}^{-1}(y(t_4)) = H_{\rm t}^{-1}(2053\,\mu\text{A}) = 293.3\,\text{K}.$

Percent stability error: $\dfrac{|293.1\,\text{K} - 293.3\,\text{K}|}{293.1\,\text{K}} = 0.06824\%.$

Calibration Error

Use measurements at t_4 and t_5.

$$\mathcal{I} = H_{\mathrm{t}}^{-1}(y(t_4)) = H_{\mathrm{t}}^{-1}(2053\,\mu\mathrm{A}) = 293.3\,\mathrm{K}.$$

$$\mathcal{A} = H_{\mathrm{t}}^{-1}(y(t_5)) = H_{\mathrm{t}}^{-1}(2040\,\mu\mathrm{A}) = 291.4\,\mathrm{K}.$$

Percent calibration error: $\dfrac{|293.3\,\mathrm{K} - 291.4\,\mathrm{K}|}{293.3\,\mathrm{K}} = 0.6478\%.$

Miscellany

Typically, error specially defined for each type of transducer.

The definition includes the exact test circuit and test conditions.

Error measures can be applied to conditioning circuits and anything else that transforms a process variable value.