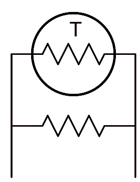
Problem 1

The temperature difference between position 1 and position 2 is to be found. Let $x_1 \in [290\,\mathrm{K}, 310\,\mathrm{K}]$ denote the temperature at position 1 and $x_2 \in [290\,\mathrm{K}, 310\,\mathrm{K}]$ denote the temperature at position 2. Temperature at position 1 is to be measured using an integrated temperature sensor with model function $H_{\mathrm{t-ITS}}(x) = x\frac{\mu\mathrm{A}}{\mathrm{K}}$ and temperature at position 2 is to be measured using a thermistor with model function $H_{\mathrm{t-thm}}(x) = R_0 e^{\beta/x}$, where $R_0 = 0.075\,\Omega$ and $\beta = 3000\,\mathrm{K}$.

(a) Show the linear form of the thermistor model function for the given temperature range in which the thermistor is placed in parallel with a linearizing resistor.

Thermistor with linearizing resistor: 

Formula (from Set 7): $\boxed{H_{\mathrm{t4}}(x) = \frac{R_\mathrm{M}}{2}\left(1 + \frac{\alpha}{2}(x - T_\mathrm{M})\right)}$,

where R_M is resistance at center of temperature range

and α is related to slope at center of temperature range.

Call center temp, T_M. From problem statement $\boxed{T_\mathrm{M} = 300\,\mathrm{K}}$.

Using (provided) exponential thermistor model:

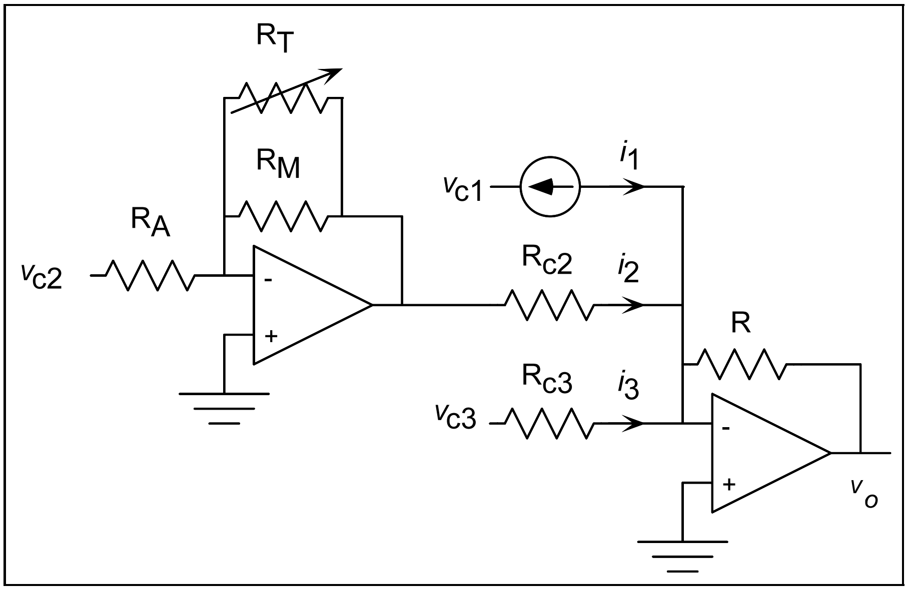
$$\boxed{R_\mathrm{M} = R_0 e^{\frac{\beta}{T_\mathrm{M}}} = 1652\,\Omega}.$$

From Set 7:

$$\boxed{\alpha = -\frac{\beta}{T_\mathrm{M}^2} = -0.03333/\,\mathrm{K}}.$$

Using the linear thermistor model function obtained above, design a circuit to convert the temperature difference $\Delta x = x_1 - x_2$ *to voltage* $H(\Delta x) = \Delta x \frac{\mathrm{V}}{2\,\mathrm{K}}$. *Show all component and source values.*

Use a summing amp since $x_1 - x_2$ is needed.



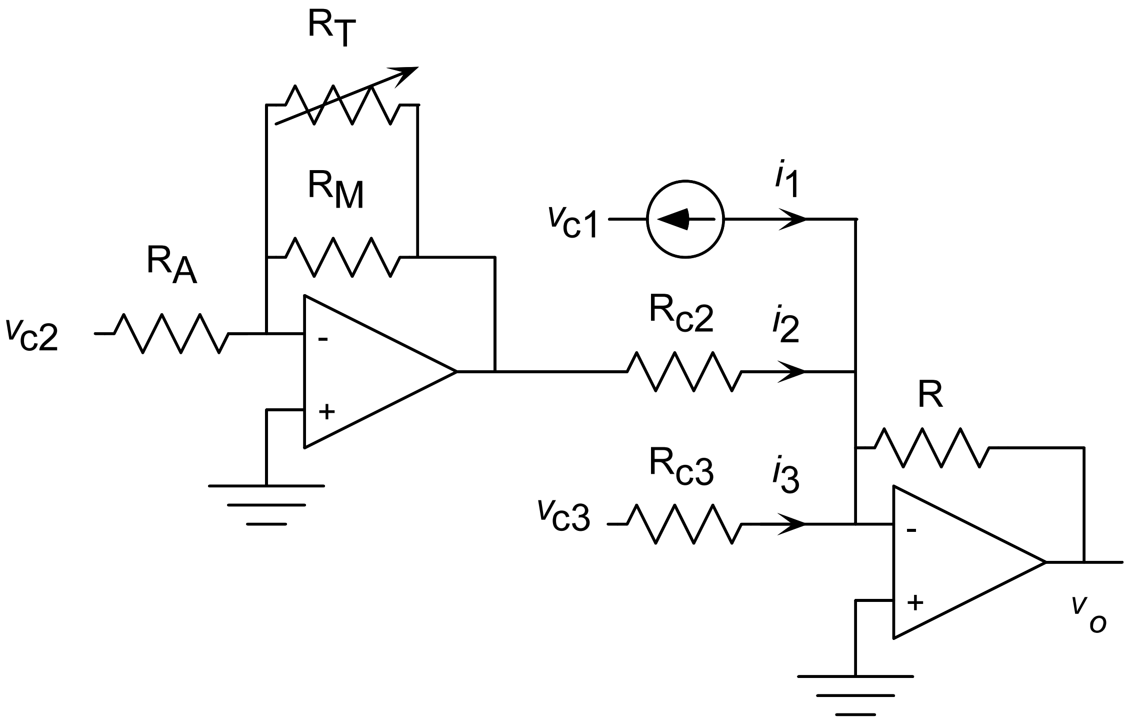
Observations:

- i_1 is a simple linear function of x_1.
- i_2 is a linear function of x_2 *plus a constant current.* (See the thermistor's linear model function.)
- A 1 Kelvin change in x_1 causes $1\,\mu$A change in i_1.
- A 1 Kelvin change in x_2 causes $z\,\mu$A change in i_2.

Solution Goals:

- Choose R_{c2}, R_{A}, and v_{c2} so $z = 1$.
- Choose R_{c3} and v_{c3} to cancel constant current.
- Choose v_{c1} to bias ITS.
- Choose R to scale output to $H(x)$.

First, find currents.



ITS current:

$$i_1 = -x_1 \frac{\mu\text{A}}{\text{K}}.$$

Thermistor current:

$$\begin{aligned} i_2 &= -v_{\text{c2}} \frac{1}{R_{\text{A}}} \frac{R_{\text{M}}}{2} \left(1 + \frac{\alpha}{2}(x_2 - T_{\text{M}})\right) \frac{1}{R_{\text{c2}}} \\ &= -i_{\text{c}} - i_{\text{c}} \frac{\alpha}{2} x_2 + i_{\text{c}} \frac{\alpha}{2} T_{\text{M}} \end{aligned}$$

where $i_{\text{c}} = \dfrac{v_{\text{c2}} R_{\text{M}}}{2 R_{\text{c2}} R_{\text{A}}}$.

Offset-resistor current:

$$i_3 = \frac{v_{\text{c3}}}{R_{\text{c3}}}.$$

So that $z = 1$ solve

$$\begin{aligned} i_1 &= i_\mathrm{c}\frac{\alpha}{2}x_2 \\ -x_1\frac{\mu\mathrm{A}}{\mathrm{K}} &= i_\mathrm{c}\frac{\alpha}{2}x_2 \\ -x_1\frac{\mu\mathrm{A}}{\mathrm{K}} &= \frac{v_{\mathrm{c}2}R_\mathrm{M}}{2R_{\mathrm{c}2}R_\mathrm{A}}\frac{\alpha}{2}x_2 \end{aligned}$$

Setting $x_1 = x_2 = 1$ and solving for $R_{\mathrm{c}2}$ yields

$$\boxed{R_{\mathrm{c}2} = \frac{v_{\mathrm{c}2}R_\mathrm{M}\alpha}{4R_\mathrm{A}}\frac{\mathrm{K}}{\mu\mathrm{A}}}.$$

(Note that both α and $v_{\mathrm{c}2}$ are negative.)

Component Choices

$v_{\mathrm{c}2}$ must be negative to get correct sign on x_2.

Arbitrary: $\boxed{v_{\mathrm{c}2} = -10\,\mathrm{V} \text{ and } R_\mathrm{A} = 1\,\mathrm{k}\Omega}$.

Using derived formula: $\boxed{R_{\mathrm{c}2} = 137.7\,\mathrm{k}\Omega}$.

To remove constant parts of thermistor current solve

$$\frac{v_{c3}}{R_{c3}} + v_{c2}\left(\frac{R_M\alpha}{4R_A}T_M - \frac{R_M}{2R_A}\right)\frac{1}{R_{c2}} = 0$$

for R_{c3} :

$$\boxed{R_{c3} = \frac{v_{c3}R_{c2}}{v_{c2}\left(\frac{R_M\alpha}{4R_A}T_M - \frac{R_M}{2R_A}\right)}}.$$

Component Values

Arbitrarily choose $\boxed{v_{c3} = 10\,\mathrm{V}}$.

Using formula, $\boxed{R_{c3} = 27.8\,\mathrm{k}\Omega}$.

ITS Bias Voltage

Most integrated temperature sensors need at lease several volts bias.

Since voltage used elsewhere, choose $\boxed{v_{c1} = 10\,\mathrm{V}}$.

Summing amplifier output after simplification:

$$v_o = -R\left(x_2\frac{\mu\mathrm{A}}{\mathrm{K}} - x_1\frac{\mu\mathrm{A}}{\mathrm{K}}\right)$$

To get desired response solve:

$$v_o = -R\left(x_2\frac{\mu\mathrm{A}}{\mathrm{K}} - x_1\frac{\mu\mathrm{A}}{\mathrm{K}}\right) = H(x_1 - x_2) = (x_1 - x_2)\frac{\mathrm{V}}{2\,\mathrm{K}}$$

$$\boxed{R = \frac{(x_1 - x_2)\frac{\mathrm{V}}{2\,\mathrm{K}}}{-(x_2 - x_1)\frac{\mu\mathrm{A}}{\mathrm{K}}}}$$

$\boxed{R = 500\,\mathrm{k}\Omega}$.

Problem 2

Design a circuit to convert force $x \in [0\,\mathrm{N}, 0.3\,\mathrm{N}]$ to a floating-point number $H(x) = x/\mathrm{N}$ to be written into variable `force`. *Use a large-displacement force sensor constructed from a non-ideal spring with response (mapping from force to amount of compression) $H_{\mathrm{spring}}(x) = 7x\frac{\mathrm{cm}}{\mathrm{N}} - 10x^2\frac{\mathrm{cm}}{\mathrm{N}^2}$ for $x \in [0\,\mathrm{N}, 0.3\,\mathrm{N}]$. Include an appropriate displacement sensor, show the function to be performed by the interface routine, and show all component and supply values.*

Range of compression: $[0, 12\,\mathrm{mm}]$.

Range of force equivalent to: $[0, 30.6\,\mathrm{g}]$.

Therefore, use a low friction displacement sensor.

CDT low friction and easiest since output in digital form.

Use CDT with 12 mm range and 8-bit precision.

$$\begin{aligned} r &= H_{\mathrm{CDT}}(H_{\mathrm{spring}}(x)) \\ &= \frac{7x\frac{\mathrm{cm}}{\mathrm{N}} - 10x^2\frac{\mathrm{cm}}{\mathrm{N}^2}}{12\,\mathrm{mm}}\left(2^b - 1\right) \\ &= 7x\frac{\mathrm{cm}}{\mathrm{N}} - 10x^2\frac{\mathrm{cm}}{\mathrm{N}^2}K \end{aligned}$$

where $b = 8$ and $K = (2^b - 1)/(12\,\mathrm{mm})$.

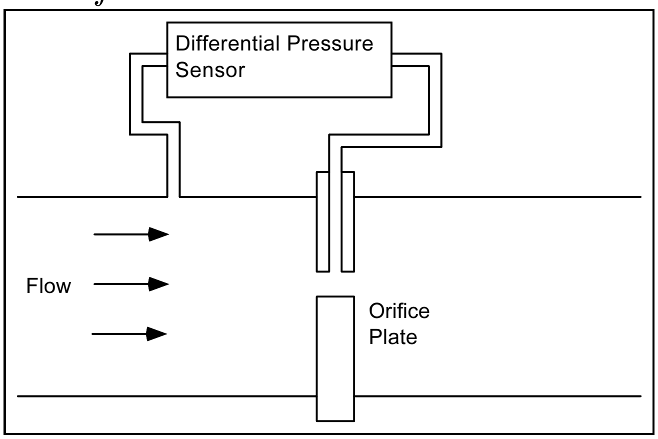
Solving for x:

$$\boxed{x = \frac{7\frac{\mathrm{cm}}{\mathrm{N}}K \mp \sqrt{\left(7\frac{\mathrm{cm}}{\mathrm{N}}K\right)^2 - 40\frac{\mathrm{cm}}{\mathrm{N}^2}Kr}}{20\frac{\mathrm{cm}}{\mathrm{N}^2}K}}$$

$\boxed{\text{Choose negative version}}$ (since x should increase with increasing r).

Problem 3(a)

Draw a diagram of an orifice plate flow sensor and explain how it works. Show any other sensors used with the orifice plate to convert the flow rate into an electrical quantity. The model function does not have to be shown.



Speed of flow increases when it passes through the orifice plate hole. Pressure is lower with a higher flow rate. The pressure near and away from the restriction is measured using a differential pressure sensor. The pressure difference is used to deduce the flow rate.

Problem 3(b)

Problem 3(b) *Explain how an ideal metal strain gauge works. In particular, explain how the electrical quantity changes with strain.*

A card with a serpentine strip of metal is glued to the object. Strain, change in shape, of the object causes the metal strip to stretch or compress. An ideal metal has constant volume and resistivity and so the strip's resistance, $\rho L/A$, will change linearly with strain, where ρ is the metal resistivity, L is its length along the direction of strain, and A is its area normal to the direction of strain.

Problem 3(c)

A strain gauge attached to a beam has model function $H_t(x) = R_0(1+2x)$*, where* $R_0 = 100\,\Omega$*. Measurements are made at four times as indicated in the table below,*

Time	Strain	Resistance/ Ω
t_1	1.3×10^{-5}	100.00237
t_2	1.3×10^{-5}	100.00257
t_3	5.1×10^{-5}	100.01000
t_4	1.3×10^{-5}	100.00297

where $t_1 < t_2 < t_3 < t_4$*, strain remains constant from* t_1 *to* t_2*, strain is measured using a calibrated device (not the strain gauge being tested), and resistance is the resistance of the strain gauge being tested. Find the model error, repeatability error, and stability error. Express each as a percent error and indicate which measurement times are used for each.*

Step 1: Invert function to get strain from resistance:

$$H_t^{-1} = \frac{1}{2}\left(\frac{R}{R_0} - 1\right).$$

Step 2: Compute strain as determined by strain gauge's resistance:

Time	Strain	Resistance/ Ω	Computed Strain
t_1	1.3×10^{-5}	100.00237	1.185×10^{-5}
t_2	1.3×10^{-5}	100.00257	1.285×10^{-5}
t_3	5.1×10^{-5}	100.01000	5.000×10^{-5}
t_4	1.3×10^{-5}	100.00297	1.485×10^{-5}

Step 3: Compute errors using definitions:

Model Error: Use actual and computed strain at t_1:

$$\frac{1.300 \times 10^{-5} - 1.185 \times 10^{-5}}{1.300 \times 10{-}5} = \boxed{8.84\%}$$

Repeatability Error: Use computed strain at t_1 (or t_2) and t_4:

$$\frac{1.185 \times 10^{-5} - 1.485 \times 10^{-5}}{1.185 \times 10{-}5} = \boxed{25.2\%}$$

Stability Error: Use computed strain at t_1 and t_2:

$$\frac{1.185 \times 10^{-5} - 1.285 \times 10^{-5}}{1.185 \times 10{-}5} = \boxed{8.44\%}$$