Homework 5, Problem 1, Solution

Find stage i link j in terms of request (a, α):

$$a_{(n-2-i)}a_{(n-3-i)} \cdots a_{(0)}\alpha_{(n-1)}\alpha_{(n-2)} \cdots \alpha_{(n-i)}a_{(n-1-i)}$$

for $0 \leq i < n-1$.

To avoid this link change destination to α' such that

$$\alpha'_{(n-1:n-i)} \neq \alpha_{(n-1:n-i)}$$

(Because there is no mechanism to re-route requests at the inputs, the changing of $a_{(n-2-i:0)}$ is *verboten*.)

In stage $n-1$ the re-routed request will be in cell:

$$\alpha'_{(n-1)}\alpha'_{(n-2)} \cdots \alpha'_{(1)}.$$

To minimize the number of auxiliary links taken set:

$\alpha'_{(n-1:n-i+1)} = \alpha_{(n-1:n-i+1)}$ and

if $\alpha_{(n-i-1)} > m/2$,

$\alpha'_{(n-i)} = \alpha_{(n-i)} + 1 \bmod m$,

$\alpha'_{(n-i-1:1)} = 0$ or

if $\alpha_{(n-i-1)} \leq m/2$,

$\alpha'_{(n-i)} = \alpha_{(n-i)} - 1 \bmod m$,

$\alpha'_{(n-i-1:1)} = m-1$.

Homework 5, Problem 2, Solution

The CGM of a $(n, m, m^{n-1}, \mathrm{S}, \mathrm{T})$ is given by:

$$I = \{ \langle 0,\, i \rangle \mid 0 \le i < m^n \}$$

$$O = \{ \langle n,\, i \rangle \mid 0 \le i < m^n \}$$

$$V = \{ \langle j,\, i \rangle \mid 0 \le i < m^n,\ 0 \le j \le n \}$$

$$E = \{ \left(\left\langle j,\, i_{(n-1)} i_{(n-2)} \ldots i_{(0)} \right\rangle, \left\langle j+1,\, i_{(n-2)} i_{(n-3)} \ldots i_{(0)} d \right\rangle \right) \mid 0 \le i < m^n,\ 0 \le j \le n-1,\ 0 \le d < m \}$$

Note that it's not necessary that input links have labels $\langle \mathrm{I},\, i \rangle$ and that output links have labels $\langle \mathrm{O},\, i \rangle$. In this case, using numbers instead of "I" and "O" simplifies the graph specification.