Name ______________________________

Interconnection Networks
EE 7725
Final Examination

16 December 1994, 15:00–17:00 CST (21:00–23:00 UTC)

Problem 1	________	(30 pts)
Problem 2	________	(20 pts)
Problem 3	________	(20 pts)
Problem 4	________	(30 pts)
Exam Total	________	(100 pts)

Good Luck!

Problem 1: The m, n χ network has the following LGM description:

$$I_\chi = \{\, \langle \mathrm{I},\, i\rangle \mid 0 \le i < mk \,\} \qquad O_\chi = \{\, \langle \mathrm{O},\, i\rangle \mid 0 \le i < mk \,\}$$

$$V_\chi = I_\chi \cup O_\chi \cup \{\, \langle x,\, i\rangle \mid 0 \le x < n,\ 0 \le i < k \,\}$$

$$\begin{aligned} E_\chi = &\{\, (\langle \mathrm{I},\, i\rangle, \langle 0,\, \chi_0(i) \bmod k\rangle) \mid 0 \le i < mk \,\} \cup \\ &\{\, (\langle 2x,\, i\rangle, \langle 2x+1,\, \lfloor \chi_1(im+d)/m \rfloor\rangle) \mid\ 0 \le d < m,\ 0 \le i < k,\ 0 \le x \le \lfloor (n-2)/2 \rfloor \,\} \cup \\ &\{\, (\langle 2x+1,\, i\rangle, \langle 2x+2,\, \lfloor \chi_0(im+d)/m \rfloor\rangle) \mid\ 0 \le d < m,\ 0 \le i < k,\ 0 \le x \le \lfloor (n-3)/2 \rfloor \,\} \cup \\ &\{\, (\langle n-1,\, i\rangle, \langle \mathrm{O},\, mi+j\rangle) \mid 0 \le j < m,\ 0 \le i < k \,\} \end{aligned}$$

where $k = m^{n-1}$, n is even, $\chi_0(i) = i_{(n-2)} i_{(n-1)} i_{(n-4)} i_{(n-3)} \ldots i_{(4)} i_{(5)} i_{(2)} i_{(3)} i_{(0)} i_{(1)}$, and $\chi_1(i) = i_{(0)} i_{(n-3)} i_{(n-2)} i_{(n-5)} i_{(n-4)} \ldots i_{(6)} i_{(3)} i_{(4)} i_{(1)} i_{(2)} i_{(n-1)}$. (In other words, $\chi_0(i)$ swaps digit positions 0 and 1, 2 and 3, …, while $\chi_1(i)$ swaps digit positions 0 and $n-1$, 1 and 2, 3 and 4, ….)

(a) Draw such a network for $n = 4$ and $m = 2$. (10 pts)

(b) Show that the m, n χ network is not terminal equivalent to the $(n, m, m^{n-1}, \mathrm{S}, \mathrm{T})$ omega network, where m is a positive integer and n is a positive even integer. (5 pts)

(c) Show that the χ network is topologically equivalent to the omega network. Show the mapping functions used. (15 pts)

The following two functions may be useful:

$$\rho_{\text{even}}(i) = i_{(n-1)} i_{(0)} i_{(n-3)} \cdots i_{(3)} i_{(n-4)} i_{(1)} i_{(n-2)}$$

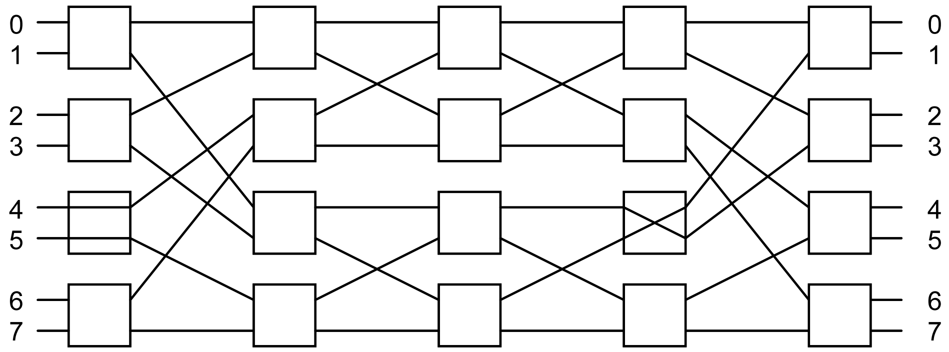
$$\rho_{\text{odd}}(i) = i_{(1)} i_{(n-2)} i_{(3)} \cdots i_{(n-3)} i_{(2)} i_{(n-1)} i_{(0)}$$

In words, $\rho_{\text{even}}(i)$ reverses the position of all even-numbered digit positions in i and $\rho_{\text{odd}}(i)$ reverses the position of all odd-numbered digit positions in i.

Problem 2: The Beneš network illustrated below is faulty. Those cells shown set are, in fact, stuck in those positions.

(a) Route the permutation connection assignment shown below. (Leave the stuck cells as they are.) Show the permutations, π_1 and π_2, for the two smaller networks. (10 pts)

$$\begin{pmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 5 & 6 & 1 & 3 & 4 & 7 & 0 & 2 \end{pmatrix} \qquad \pi_1 = \begin{pmatrix} 0 & 1 & 2 & 3 \\ & & & \end{pmatrix} \qquad \pi_2 = \begin{pmatrix} 0 & 1 & 2 & 3 \\ & & & \end{pmatrix}$$



(b) What is the maximum number of such stuck-at failures that a 2^n-input Beneš network could tolerate while still being able to realize all permutation connection assignments? Answer the question assuming 1) the faults could occur anywhere and 2) the faults are restricted to parts of the network (specify the parts). Briefly justify your answers. (10 pts)

Problem 3: A direct network is to be constructed from a $(n, m, m^{n-1}, \mathrm{S}, \mathrm{T})$ omega network by connecting inputs to like-numbered outputs (*i.e.*, input 0 to output 0, input 1 to output 1, etc.) and by making the links bi-directional. (A message can pass either way though the links, as is the case for the k-ary n-cube, deBruijn network, and other direct networks presented in class.) As with all the direct networks studied in class, each cell is connected to a processor through an additional link.

(a) What is the number of nodes, diameter, and degree for this network? Show two nodes whose distance apart is the network diameter. (10 pts)

(b) Explain how a request might be routed in this network. (The answer does not have to find the shortest path.) (10 pts)

Problem 4: Answer each of the questions below. Be brief.

(a) Can two networks be proven DE (descriptively equivalent) (or not) based on their LGM? Explain. (6 pts)

(b) Find all routing tags for request $(123, 240)$ in a $(6, 3, 82, \mathrm{S}, \mathrm{T})$ modified omega network. (6 pts)

It's not over yet, more problems on the next page.

(c) A bus is being considered for the interconnection in a parallel computer. It will be evaluated using the performance model presented early in the semester (see Note Set 1). Let $t_{\mathrm{nw-bus}}$ denote the latency of messages in the bus. For what relative values of parameters N, P, and t_m would a bus be a good choice? Explain. (6 pts)

(d) Either a direct or multistage network will be chosen for the interconnection in a parallel computer. They will be evaluated using the performance model presented early in the semester (and used in the previous short-answer problem). The latency of the multistage network is higher than the direct network. The throughput of the two networks is about the same. Suppose the direct network were more expensive. For what relative values of the parameters is the multistage network better and for what relative values of the parameters is the direct network better? Cost is a factor. Explain. (6 pts)

(e) Describe a situation in which a direct network would have lower latency than a multistage network, even at high traffic volumes. (6 pts)

Have a good holiday!