Problem 3: A direct network is to be constructed from a $(n, m, m^{n-1}, \mathrm{S}, \mathrm{T})$ omega network by connecting inputs to like-numbered outputs (*i.e.*, input 0 to output 0, input 1 to output 1, etc.) and by making the links bi-directional. (A message can pass either way though the links, as is the case for the k-ary n-cube, deBruijn network, and other direct networks presented in class.) As with all the direct networks studied in class, each cell is connected to a processor through an additional link.

(a) What is the number of nodes, diameter, and degree for this network? Show two nodes whose distance apart is the network diameter. (10 pts)

Easy part: $\boxed{\text{Number of nodes, } nm^{n-1}.}$ $\boxed{\text{Degree, } 2m.}$

Hard part: diameter. To find the diameter two nodes will be found that are as far apart as possible. Label the stage-x, cell-i node $\langle x, i\rangle$. Note that i is a $(n-1)$-digit radix-m number. Key observation: to move from one node to another node in which each digit in the cell number is different, each stage must be visited. Therefore the cell number of the two nodes should differ in every digit, for example $\langle x, 00\cdots 0\rangle$ and $\langle y, 11\cdots 1\rangle$; call these node X and node Y. The nodes are at least distance $n-1$ apart (for example when $x=0$ and $y=n-1$). They can be chosen to be further apart by making x and y different. Without loss of generality, let $x=0$. A message en route from X to Y, right after all the digits in the cell number are changed to 1's will either be in stage 1 or stage $n-1$ (depending upon which direction it starts). When $n>3$ the most distant stage from both 1 and $n-1$ is $\lfloor n/2\rfloor$. Therefore, the diameter when $n>3$ is $\boxed{n-1+\lfloor n/2\rfloor-1=\lfloor 3n/2\rfloor-2}$ and two furthest apart nodes are $\boxed{\langle 0, 0\rangle}$ and $\boxed{\langle\lfloor n/2\rfloor, 11\cdots 1\rangle.}$ When $n=2$ set $y=0$ for a diameter of $1+1=2$.

(b) Explain how a request might be routed in this network. (The solution does not have to use the shortest path.) (10 pts)

Call the destination $\langle x, j\rangle$.

Step 1: Route to stage $x-n+1 \bmod n$ using arbitrary forward or backward links, whichever are faster.

Step 2: In stage y use forward cell output $j_{((x-y-1)\bmod n)}$.