Name ________________________________

Interconnection Networks
EE 7725
Final Examination

14 December 1995, 10:00–12:00 C. S. T.

Modified.

Problem 1	________	(25 pts)
Problem 2	________	(30 pts)
Problem 3	________	(20 pts)
Problem 4	________	(25 pts)
Exam Total	________	(100 pts)

Good Luck!

Problem 1: A *generalized butterfly network* is a generalization of a butterfly network in the same way a generalized omega network is a generalization of the omega network. A $(m_0, m_1, \ldots, m_{n-1})$-GB network is a generalized butterfly using $m_0 \times m_0$ cells in the first stage, $m_1 \times m_1$ cells in the second stage, and so on.

(*a*) Draw a $(2,3,2)$-GB network. (10 pts)

(*b*) Find the sequence of cells used to route request $(232, 741)$ in a $(7,3,8,5)$-GB network. The cell numbers may be specified as mixed-radix numbers. Be sure to specify the radix for each number. The table of decimal to mixed radix conversions may be helpful. (15 pts)

Radix MSD,...,LSD	232 MSD,...,LSD	741 MSD,...,LSD
7, 3, 8, 5	1, 2, 6, 2	6, 0, 4, 1
7, 3, 5, 8	1, 2, 4, 0	6, 0, 2, 5
7, 8, 3, 5	1, 7, 1, 2	6, 1, 1, 1
7, 8, 5, 3	1, 7, 2, 1	6, 1, 2, 0
7, 5, 3, 8	1, 4, 2, 0	6, 0, 2, 5
7, 5, 8, 3	1, 4, 5, 1	6, 0, 7, 0
3, 7, 8, 5	0, 5, 6, 2	2, 4, 4, 1
3, 7, 5, 8	0, 5, 4, 0	2, 4, 2, 5
3, 8, 7, 5	0, 6, 4, 2	2, 5, 1, 1
3, 8, 5, 7	0, 6, 3, 1	2, 5, 0, 6
3, 5, 7, 8	0, 4, 1, 0	2, 3, 1, 5
3, 5, 8, 7	0, 4, 1, 1	2, 3, 1, 6
8, 7, 3, 5	2, 1, 1, 2	7, 0, 1, 1
8, 7, 5, 3	2, 1, 2, 1	7, 0, 2, 0
8, 3, 7, 5	2, 0, 4, 2	7, 0, 1, 1
8, 3, 5, 7	2, 0, 3, 1	7, 0, 0, 6
8, 5, 7, 3	2, 1, 0, 1	7, 0, 2, 0
8, 5, 3, 7	2, 1, 0, 1	7, 0, 0, 6
5, 7, 3, 8	1, 2, 2, 0	4, 2, 2, 5
5, 7, 8, 3	1, 2, 5, 1	4, 2, 7, 0
5, 3, 7, 8	1, 1, 1, 0	4, 1, 1, 5
5, 3, 8, 7	1, 1, 1, 1	4, 1, 1, 6
5, 8, 7, 3	1, 3, 0, 1	4, 3, 2, 0
5, 8, 3, 7	1, 3, 0, 1	4, 3, 0, 6

Problem 2: A *ranker* is a device with N 1-bit inputs and N $\lceil \log_2 N \rceil$-bit outputs. As usual, let the inputs and outputs be numbered 0 to $N-1$. A sequence of 0's and 1's is placed at the inputs. The ranker puts x at output i iff there are exactly x 1's placed at inputs 0 to $i-1$. Output 0 is always has a 0. (Output 0 isn't really necessary. It's only there to make the problem description slightly simpler.) For example if sequence 0,1,1,0,1 were at the inputs, then sequence 0,0,1,2,2 would be at the outputs.

(a) Explain how a ranker and a banyan could be used to build a device that sorts 1-bit keys along with a data item associated with each key. Specify which banyan to use, any non-standard features of the cells, and any other hardware needed. *Hint: The solution is based on one of the permutation families.* (12 pts)

(b) Can one conclude from the zero-one theorem that this network sorts all sequences? (8 pts)

(c) Explain how to construct a self-routing permutation network using these (or any other) 1-bit sorters. (10 pts)

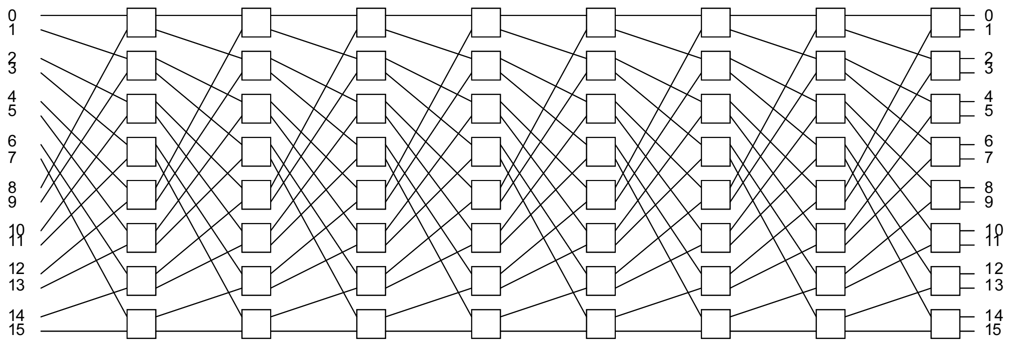
Problem 3: A permutation is a *complete J-partition* if it is obtained by recursively J-partitioning an omega network into two parts until partitions consist of single cells. Let $\mathcal{C}$ denote the set of all complete J-partitions of a 2^n-input omega network. Show that $\Lambda \subseteq \mathcal{C}$ and $\Upsilon \subseteq \mathcal{C}$, that is, the lower- and upper-triangular permutations are a subset of the set of all complete J-partitions. (20 pts)

Problem 4: Answer each problem below.

(a) As was shown in class, the cells in the input and output stages of a $(3,(2^{\mu},k,2^{\mu}),(k,2^{\mu},k),\mathrm{T},\mathrm{T})$ Clos network need not realize all permutations. If these cells were implemented using crossbars, and each cell had to be identical, how many crosspoints would be saved? Explain. (5 pts)

(b) As we all know, the cells in a *d-dilated* network have $2d$ inputs and $2d$ outputs. A naïve implementation would use $(2d)^2$ crosspoints. The alert engineer might realize that a significant number of crosspoints can be eliminated if each cell input in the upper set connects to exactly one of the upper set of outputs (its counterpart, perhaps). Complete the idea, draw a diagram of the cell, and compute the number of crosspoints. The less-expensive cell should be able to handle any conflict that the naïve cell could.(5 pts)

(c) One futile approach to proving that an omega-omega network ($2n$ shuffle stages) can realize all permutations is to show that it is topologically equivalent to a Beneš network (ignoring one of the stages in the omega-omega network). Prove that this approach is futile (that the omega-omega is not topologically equivalent to a Beneš network when ignoring a stage). The 16-input omega-omega network below can be used to help explain the proof. *Hint 1: A mapping function is not necessary. Hint 2: The proof can be quickly explained. Hint 3: The proof must consider the center stages of the network.* (5 pts)



This paragraph is part of the next two problems. In the network using feedback to limit the size of saturation trees, a damping method was employed to prevent a saturation tree from reforming at outputs which undergo a hot-to-cold transition.

(d) Explain how the damping method works. (5 pts)

(e) Suppose a similar feedback method were used in k-ary n-cubes, but without the damping mechanism. Would the size (number of requests) in a re-formed saturation tree be a function of k (for a fixed N)? Explain. (5 pts)

Have a good vacation.