EE 7725 **Homework 5** **Solution**

Solution 1: Consider request (a, α), part of a bitonic permutation, as it passes through a stage x cell in an omega network. It will enter using link $a_{(n-x-2)} \cdots a_{(0)} \alpha_{(n-1)} \cdots \alpha_{(n-x)} a_{(n-x-1)}$ and exit through $a_{(n-x-2)} \cdots a_{(0)} \alpha_{(n-1)} \cdots \alpha_{(n-x)} \alpha_{(n-x-1)}$. The destinations of the $m-1$ other requests passing though the cell will have the same $x-1$ most-significant digits, $\alpha_{(n-x)}\alpha_{(n-x)}$. Digit $\alpha_{(n-x-1)}$ is "added" by having them use the corresponding cell output; because bitonic permutations are admissible, the outputs will be distinct. Because the most significant digits are present and in order in the link numbers, the requests at the output of a cell are sorted in the same relative order as they will appear at the network output. Therefore, assuming requests were properly routed to stage x cell inputs, they could be routed to stage-x cell outputs by sorting the corresponding integers in the bitonic sequence (which would be part of the message). The routing hardware would then have to sort m integers rather then examine a single digit in the destination. Since, by a similar argument, in stage 0 the requests will be properly routed, by induction they will be properly routed in all stages.

Sorting m integers is more complex than examining m digits, but as long as m is small the sort can easily be done. Note that this network, called a bitonic sorter, does much more than an omega network routing a bitonic permutation. The rank (*e.g.*, first, fifth, last) of each element of a bitonic permutation is given whereas in a bitonic sequence there is no way of telling where in the sorted sequence an element will fall.

Solution 2: The network must be partitioned twice. First the network is partitioned using $J_1 = \{0\}$ which places odd and even inputs and outputs into separate partitions. The even inputs are called partition A, the odd inputs must be further divided. Note that the input numbers belonging to partition B, 1, 5, 9, and 13 all have a zero in digit position 2, so this position must be used to divide the odd network partition. Call the set used to specify this partition J_2. Using the host (original) network numbering, $J_2 = \{2\}$; using the numbering of the odd partition, $J_2 = \{1\}$. Either answer is considered correct. However, $J = \{0, 2\}$ is not correct since that would split partition A.

There are two X permutation, one for each J. Call them X_1 and X_2; both are the same: $X_1 = X_2 = \begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix}$.

To set the network to realize the permutation for B, set cell 3 in stage 0 to transpose, and the other B partition cells to identity.