Name ______________________________

Interconnection Networks
EE 7725
Final Examination

9 December 1996,   10:00–12:00 C. S. T.

Modified

Problem 1	________	(30 pts)
Problem 2	________	(30 pts)
Problem 3	________	(20 pts)
Problem 4	________	(20 pts)
Exam Total	________	(100 pts)

Alias ______________________________

Good Luck!

Problem 1: Consider the following network, derived from an omega network, in which cells are non-square. The network is described by $\big((m_0, m'_0), (m_1, m'_1), \ldots, (m_{n-1}, m'_{n-1})\big)$, where element (m_x, m'_x) specifies the number of inputs and outputs of cells in stage x. (That is, stage x uses cells of size $m_x \times m'_x$.) Let $k_x = \left(\prod_{i=0}^{x-1} m'_i\right)\left(\prod_{i=x+1}^{n-1} m_i\right)$. Stage x uses a σ_{m_x,k_x} link pattern before the cells (the straightforward generalization of the omega network shuffle link pattern).

(a) What are the number of links before the cells in each stage? How many outputs does the network have? (10 pts)

(b) Is this network a banyan? Is it a delta? Explain. (7 pts)

(c) For the $((2,3),(4,1),(5,6),(2,2),(7,2))$ network find the cell inputs and outputs used on the path from input 433 to output 70. The answer can be in mixed-radix form; if so specify the radix. (13 pts)

Problem 2: A permutation P of $\langle m^n \rangle$ is a radix-m, n-digit *digit permute* permutation if there exists $\pi \in \Sigma_n$ such that for all $(a, \alpha) \in P$ and $x \in \langle n \rangle$ $\alpha_{(x)} = a_{(x\pi)}$. In other words, the destination is obtained by rearranging the digits of the source as indicated by π.

The digit-reverse permutation, ρ, is a digit-permute permutation in which

$$\pi = \begin{pmatrix} 0 & 1 & \cdots & n-1 \\ n-1 & n-2 & \cdots & 0 \end{pmatrix}.$$

Most of the link patterns used in class, including the shuffle, are digit permute permutations*.

(a) Prove that there is exactly one radix-m, n-digit permutation in $\Omega_{m,n}$. (15 pts)

(b) Call a bidelta network *normally labeled* if the cell outputs used on the path to an output are the radix-m digits of the output (with the order depending on the network) and the cell inputs used on the path from an output to an input are the radix-m digits of the input (also with the order depending on the network). (The omega, baseline, butterfly, and most other networks covered in class are normally labeled.)

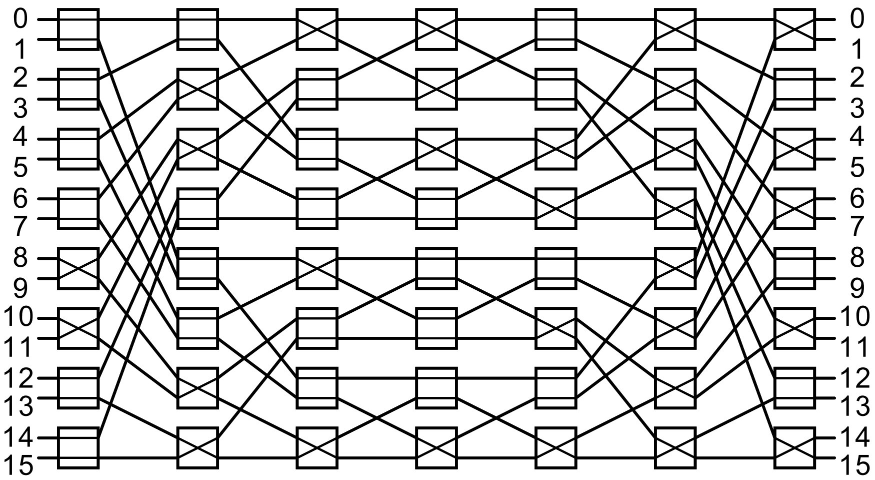
Prove that there is exactly one radix-m, n-digit permutation realizable by any normally labeled radix-m, n-stage bidelta network. (10 pts)

(c) What are the digit permute permutations realizable by the following networks: omega, baseline, and butterfly. (5 pts)

* Yes, this is from problem 1 on homework 4b.

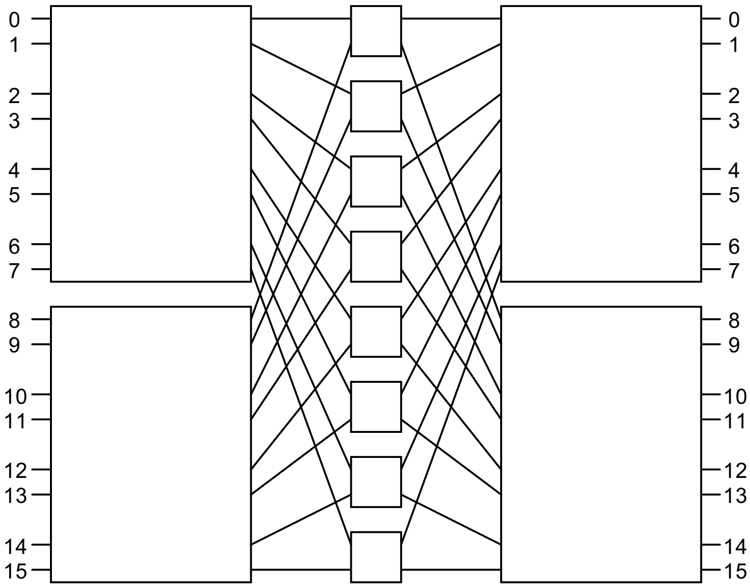
Problem 3: Consider the following permutation and its routing in the 16-input Beneš network:

$$P = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 \\ 15 & 6 & 12 & 0 & 1 & 3 & 11 & 9 & 4 & 5 & 14 & 2 & 7 & 13 & 8 & 10 \end{pmatrix}$$



(a) Show that this permutation can be routed at least 80 ways on a 16-input Beneš network. (For partial credit, show that it can be routed $y < 80$ ways for a large a y as your time permits.) (10 pts)

(b) Show the permutations for the 8×8 cells in the Clos network with $m = 8$ and $k = 2$ corresponding to the Beneš network realizing P above. (10 pts)



Problem 4: Answer each question below.

(a) In a two-level deadlock-free adaptive routing scheme, would it be okay to allow a message to go from the deadlock-free layer to the adaptive layer just once? Explain. (5 pts)

(b) Give an example of a network which is a delta network but where the path taken by a request in a stage is **not** based on a digit of the destination. *Hint: The example can be constructed with a minor modification to the omega network.* (5 pts)

(c) Find the utilization under the following version of the performance model: There are P tasks and N processors, each task computes for t_m time then performs communication which takes t_{nw} time to complete; after their communication each task computes for another t_m time, etc. Unlike the model given in class, processors can be waiting for at most c communications. That is, if c tasks in a processor are waiting for communication to complete, a new communication can not start until one of the c finishes. Derive an expression utilization; the expression should be valid for all positive c. Assume that P is divisible by N. (10 pts)