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Interconnection Networks
EE 7725
Final Examination

9 December 1996, 10:00–12:00 C. S. T.

Modified

Problem 1	_____	(30 pts)
Problem 2	_____	(30 pts)
Problem 3	_____	(20 pts)
Problem 4	_____	(20 pts)
Exam Total	_____	(100 pts)

Alias Answer Key

Good Luck!

Problem 1: Consider the following network, derived from an omega network, in which cells are non-square. The network is described by $\left((m_0, m'_0), (m_1, m'_1), \ldots, (m_{n-1}, m'_{n-1})\right)$, where element (m_x, m'_x) specifies the number of inputs and outputs of cells in stage x. (That is, stage x uses cells of size $m_x \times m'_x$.) Let $k_x = \left(\prod_{i=0}^{x-1} m'_i\right)\left(\prod_{i=x+1}^{n-1} m_i\right)$. Stage x uses a σ_{m_x,k_x} link pattern before the cells (the straightforward generalization of the omega network shuffle link pattern).

(a) What are the number of links before the cells in each stage? How many outputs does the network have? (10 pts)

Total number of links at inputs to a cell in stage x is $m_x k_x$.

Number of network outputs: $\prod_{i=0}^{n-1} m'_i$.

(b) Is this network a banyan? Is it a delta? Explain. (7 pts)

It's both a banyan and a delta. As with other omega networks, a path through the network is constructed by shuffling and exchange digits. The difference is that at stage x a radix-m_x digit is replaced by a radix-m'_x digit. There is still exactly one way to route from any input to any output and the path taken to a particular output is the same regardless of the input it starts at.

(c) For the $((2,3),(4,1),(5,6),(2,2),(7,2))$ network find the cell inputs and outputs used on the path from input 433 to output 70. The answer can be in mixed-radix form; if so specify the radix. (13 pts)

First convert 433 and 67 to the appropriate mixed-radix form. The radix for the input is found from the number of cell inputs in each stage, $(2,4,5,2,7)$ in this case, so $433_{10} = 12016$. Similarly, the output radix is $(3,1,6,2,2)$ and the output number is $67_{10} = 20510$. The path through the network is:

Stage	Radix	Link Number
Input	2 4 5 2 7	1 2 0 1 6
0 Cell In	4 5 2 7 2	2 0 1 6 2
0 Cell Out	4 5 2 7 3	2 0 1 6 2
1 Cell In	5 2 7 3 5	0 1 6 2 2
1 Cell Out	5 2 7 3 1	0 1 6 2 0
2 Cell In	2 7 3 1 5	1 6 2 0 0
2 Cell Out	2 7 3 1 6	1 6 2 0 5
3 Cell In	7 3 1 6 2	6 2 0 5 1
3 Cell Out	7 3 1 6 2	6 2 0 5 1
4 Cell In	3 1 6 2 7	2 0 5 1 6
4 Cell Out	3 1 6 2 2	2 0 5 1 0

Problem 2: A permutation P of $\langle m^n \rangle$ is a radix-m, n-digit *digit permute* permutation if there exists $\pi \in \Sigma_n$ such that for all $(a, \alpha) \in P$ and $x \in \langle n \rangle$ $\alpha_{(x)} = a_{(x\pi)}$. In other words, the destination is obtained by rearranging the digits of the source as indicated by π.

The digit-reverse permutation, ρ, is a digit-permute permutation in which

$$\pi = \begin{pmatrix} 0 & 1 & \cdots & n-1 \\ n-1 & n-2 & \cdots & 0 \end{pmatrix}.$$

Most of the link patterns used in class, including the shuffle, are digit permute permutations*.

(a) Prove that there is exactly one radix-m, n-digit permutation in $\Omega_{m,n}$. (15 pts)

The one-and-only digit permute permutation (DPP) in Ω is (drumroll) identity; it is well known that identity is in Ω. Let π be any permutation of $\langle n \rangle$ except identity. It must be shown that the corresponding DPP is not in Ω. Consider request (a, α) in the DPP; by definition $\alpha_{(i)} = a_{(i\pi)}$ for all i. Since π is not identity there must exist some $j \in \langle n \rangle$ such that $j > j\pi$. Using such a j let $x = n - 1 - j$ and consider the link used by (a, α) at the output of a cell in stage x:

$$\begin{aligned} & a_{(n-x-2)} \cdots a_{(j\pi)} \cdots a_{(0)} \alpha_{(n-1)} \cdots \alpha_{(n-x-1)} \\ = \; & a_{(n-x-2)} \cdots a_{(j\pi)} \cdots a_{(0)} \alpha_{(n-1)} \cdots \alpha_{(j)} \\ = \; & a_{(n-x-2)} \cdots a_{(j\pi)} \cdots a_{(0)} \alpha_{(n-1)} \cdots a_{(j\pi)} \end{aligned}$$

Since $n - x - 1 = j$, $\alpha_{(n-x-1)} = \alpha_{(j)} = a_{(j\pi)}$. This means that each of the m requests entering the cell (for each possible value of $a_{(n-x-1)}$) all follow a path using the one cell output link (since digits $a_{(j\pi)}$ must all be identical to reach this cell), since paths cannot share a link no non-identity DPP permutation is in Ω.

(b) Call a bidelta network *normally labeled* if the cell outputs used on the path to an output are the radix-m digits of the output (with the order depending on the network) and the cell inputs used on the path from an output to an input are the radix-m digits of the input (also with the order depending on the network). (The omega, baseline, butterfly, and most other networks covered in class are normally labeled.)

Prove that there is exactly one radix-m, n-digit permutation realizable by any normally labeled radix-m, n-stage bidelta network. (10 pts)

Every cell in a stage of a bidelta network can be uniquely labeled by its dual-path descriptor; in a normally labeled bidelta that's some subset of digits in the inputs and outputs reachable from the cell. The dual-path descriptors of reachable cells in the next stage are obtained by dropping one of the input digits and adding an output digit. Suppose the bidelta network is to realize a non-identity DPP, and let $\alpha_{(j)} = a_{(j\pi)}$ be the digit added in the reachable next stage cells. If $a_{(j\pi)}$ is present in the dual-path descriptor then every request entering the cell will have to share a link to the same next-stage cell, so the permutation cannot be realized. For all values of π but one, there must be at least one case where digit $a_{(j\pi)}$ is present, so only that DPP can be realized by a normally labeled DPP. That special π is determined by the network: if cells in stage x "replace" source digit $a_{(j)}$ with destination digit $\alpha_{(j')}$ then $j'\pi = j$.

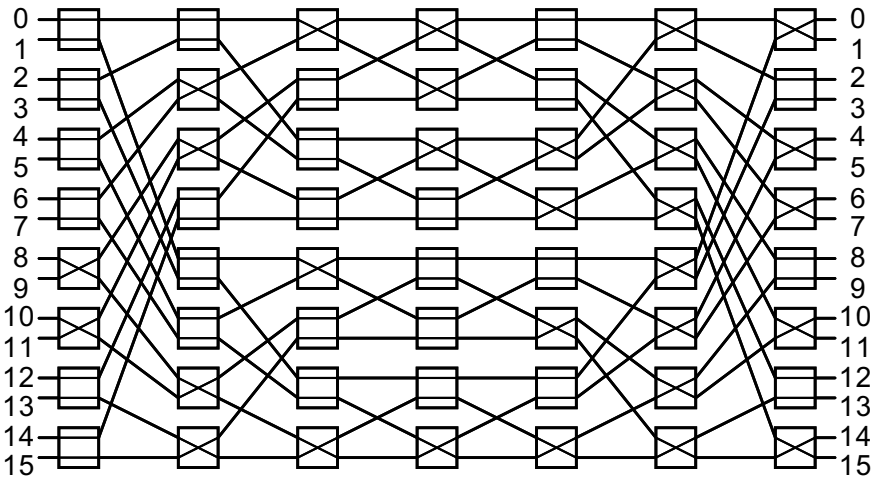
(c) What are the digit permute permutations realizable by the following networks: omega, baseline, and butterfly. (5 pts)

In the omega and inverse omega, source digit $a_{(j)}$ is replaced with destination digit $\alpha_{(j)}$ for $j \in \langle n \rangle$ so π is identity. In the baseline network source digit $a_{(j)}$ is replaced by $\alpha_{(n-1-j)}$ so $j\pi = n - j - 1$. In the butterfly $a_{(j)}$ is replaced by $\alpha_{(j-1 \bmod n)}$, so $j\pi = j + 1 \bmod n$.

* Yes, this is from problem 1 on homework 4b.

Problem 3: Consider the following permutation and its routing in the 16-input Beneš network:

$$P = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 \\ 15 & 6 & 12 & 0 & 1 & 3 & 11 & 9 & 4 & 5 & 14 & 2 & 7 & 13 & 8 & 10 \end{pmatrix}$$



(a) Show that this permutation can be routed at least 80 ways on a 16-input Beneš network. (For partial credit, show that it can be-routed $y < 80$ ways for a large a y as your time permits.) (10 pts)

Identify all of the loops (as defined by the looping algorithm) in the routed network. Each loop can be routed two ways. The top level network has three loops (starting at input 0, 6, and 8); these can be routed 8 different ways. The second-level networks have three more loops and the third level networks have 6 loops. If there are x loops in a level that level can be routed 2^x ways. Since only two routings at one level (change the state of every cell) are sure to preserve the number of loops at lower levels the alternatives are combined in the following way $2^3 - 2^1 + 2^1(2^3 - 2^2 + 2^2(2^6)) = 526$.

(b) Show the permutations for the 8×8 cells in the Clos network with $m = 8$ and $k = 2$ corresponding to the Beneš network realizing P above. (10 pts)

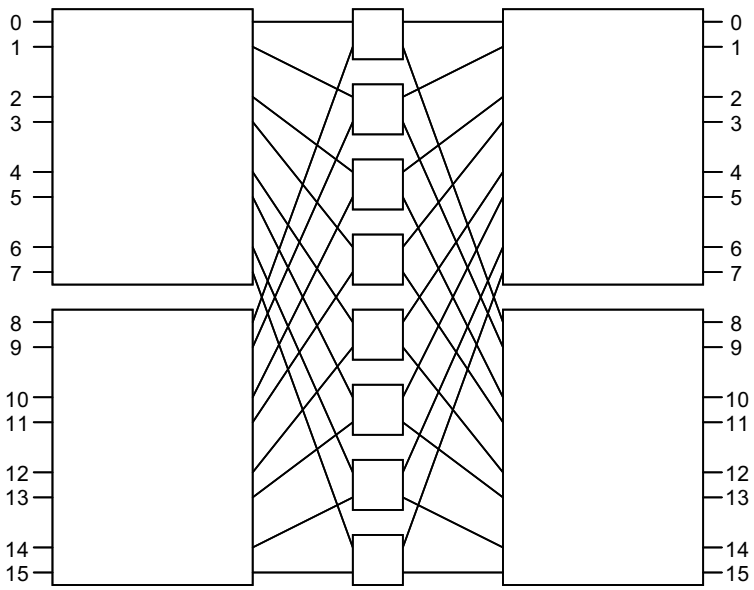
Solution:

$$P_{\langle 0,0 \rangle} = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 1 & 5 & 2 & 6 & 3 & 4 & 0 & 7 \end{pmatrix}$$

$$P_{\langle 0,1 \rangle} = \begin{pmatrix} 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 \\ 15 & 10 & 12 & 8 & 9 & 14 & 11 & 13 \end{pmatrix}$$

$$P_{\langle 1,0 \rangle} = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 2 & 7 & 5 & 1 & 3 & 6 & 0 & 4 \end{pmatrix}$$

$$P_{\langle 1,1 \rangle} = \begin{pmatrix} 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 \\ 11 & 15 & 12 & 8 & 14 & 10 & 13 & 9 \end{pmatrix}$$



Problem 4: Answer each question below.

(a) In a two-level deadlock-free adaptive routing scheme, would it be okay to allow a message to go from the deadlock-free layer to the adaptive layer just once? Explain. (5 pts)

No, a message moving from the deadlock-free to the adaptive layer might "get stuck" behind the tail of a message whose head is back in the deadlock-free layer. That head might be blocked behind the message leaving, a deadlocked situation. (Note: deadlock would not occur if buffers always had enough space for incoming networks, as in store and forward systems.)

(b) Give an example of a network which is a delta network but where the path taken by a request in a stage is **not** based on a digit of the destination. *Hint: The example can be constructed with a minor modification to the omega network.* (5 pts)

The network can be obtained from an omega network by adding one to each output label. (Some people just don't like it when things are numbered starting at zero.)

(c) Find the utilization under the following version of the performance model: There are P tasks and N processors, each task computes for t_m time then performs communication which takes t_{nw} time to complete; after their communication each task computes for another t_m time, etc. Unlike the model given in class, processors can be waiting for at most c communications. That is, if c tasks in a processor are waiting for communication to complete, a new communication can not start until one of the c finishes. Derive an expression utilization; the expression should be valid for all positive c. Assume that P is divisible by N. (10 pts)

Solution:

$$U = \begin{cases} \min\left\{\frac{ct_m}{t_{nw}}, 1\right\} & \text{if } C < \frac{P}{N} \\ \min\left\{\frac{\frac{P}{N}t_m}{t_m + t_{nw}}, 1\right\} & \text{if } C \geq \frac{P}{N} \end{cases}$$