Name Solution Key

Interconnection Networks
EE 7725
Midterm Examination

29 October 1996, 18:40–19:40 C. S. T.

Problem 1		(35 pts)
Problem 2		(35 pts)
Problem 3		(30 pts)
Exam Total		(100 pts)

Alias $\Gamma\rho\epsilon\epsilon\kappa\ \tau\omega\ \mu\epsilon$

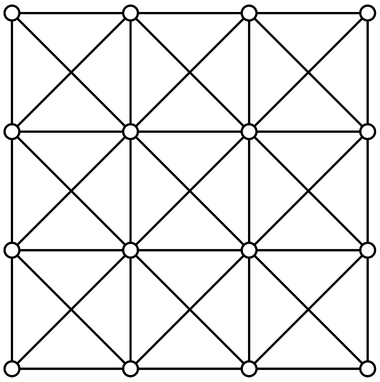
Good Luck!

Problem 1: A $k \times k$ *double mesh* network is defined by

$$\begin{aligned} V =& \{\, 0, 1, \ldots, k^2 - 1 \,\} \\ E =& \{\, (i,j) \mid i \in V,\ x \in \{0,1\},\ i_{(x)} < k-1,\ j = i + k^x \,\} \cup \\ & \{\, (i,j) \mid i \in V,\ i_{(0)} < k-1,\ i_{(1)} < k-1,\ j = i + 1 + k \,\} \cup \\ & \{\, (i,j) \mid i \in V,\ i_{(0)} < k-1,\ i_{(1)} > 0,\ j = i + 1 - k \,\}, \end{aligned}$$

where $i_{(x)}$ is digit x in the radix-k representation of i.

(a) Draw a 4×4 double mesh network. (9 pts)



(b) What are the degree and diameter of a $k \times k$ double-mesh network? (8 pts)

Degree,

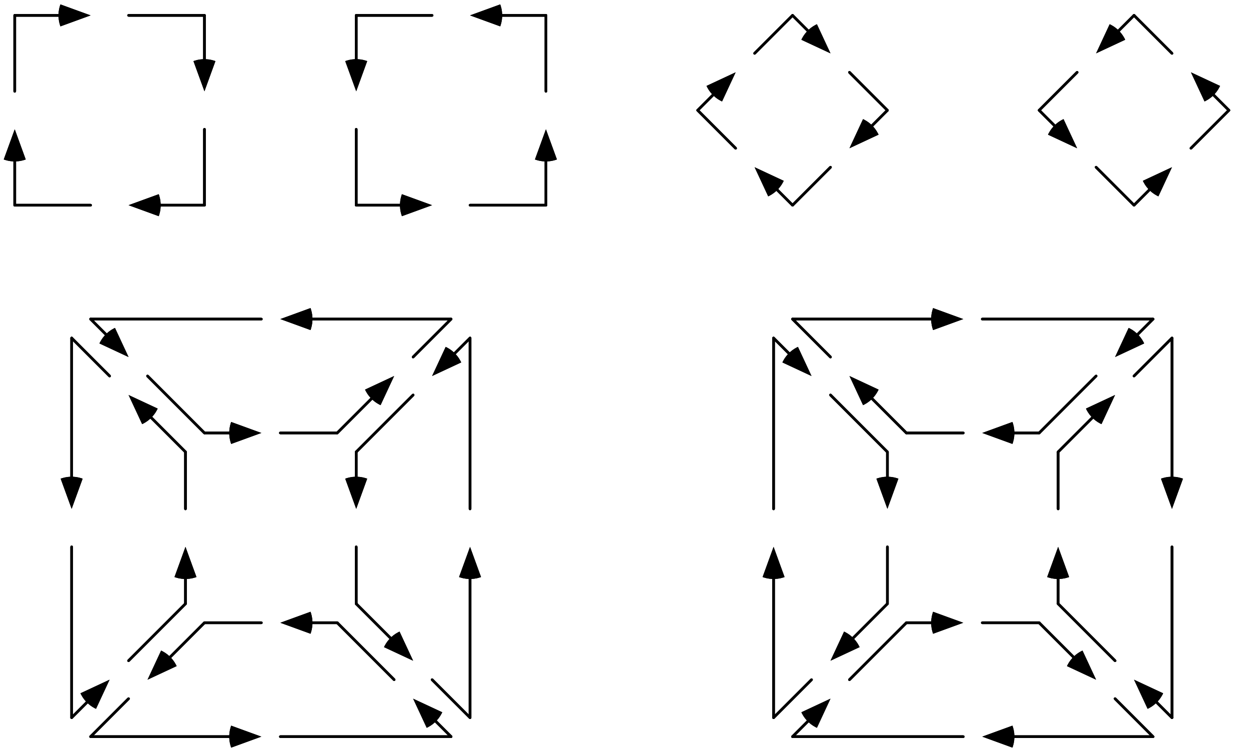
$$\delta = \begin{cases} 3, & \text{if } n = 2; \\ 8, & \text{if } n > 2. \end{cases}$$

Diameter, $D = k - 1$.

(c) Give a formula for $d_{i,j}$, the distance between $i, j \in V$. (9 pts)

Distance formula: $d_{i,j} = \max\left\{\, \left|i_{(0)} - j_{(0)}\right|, \left|i_{(1)} - j_{(1)}\right| \,\right\}$

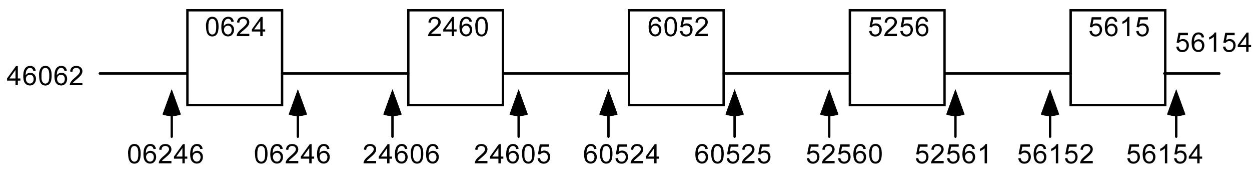
(d) Show all turns (as in the turn model) possible in this network. (9 pts)



Problem 2: For $n > 1$ an n-stage, $m \times m$ *double-shuffle network* is an m^n-input network consisting of n identical stages, each having a $\sigma_{m^2,m^{n-2}}$ link pattern followed by m^{n-1} cells of size $m \times m$. (The structure is similar to the omega network.)

(a) Find the sequence of cells and links taken by request $(11706, 14151)$ in a 5-stage, 7×7 double-shuffle network. Note, $11706_{10} = 46062_7$ and $14151_{10} = 56154_7$. (20 pts)

All numbers are in radix 7.



(b) Find a mapping function for the inputs and stage-0 cells from an n-stage, $m \times m$ double-shuffle network to an n-stage, $m \times m$ omega network, where $n > 1$. The mapping function should show that the two networks are topologically equivalent *up to and including the first stage.* (8 pts)

Mapping function:

$$f(\langle \mathrm{I},\, i \rangle) = \left\langle \mathrm{I},\, \sigma_{m,m^{n-1}}(i) \right\rangle$$
$$f(\langle 0,\, i \rangle) = \langle 0,\, i \rangle$$

(c) Show that the double-shuffle network is not topologically equivalent to an n-stage, $m \times m$ omega network, where $n > 1$. *Hint: this can easily be solved without mapping functions.* (7 pts)

If there are an even number of stages odd digit positions can not be changed, so not all outputs are reachable. To see this draw a 2-stage 2×2 double shuffle network.

Problem 3: Answer each question below.

(a) Find the shortest path from node 0123456 to 0564123 in a 10, 7 de Bruijn network. (7 pts)

Source: 0123456 $\rightarrow$ 123456x $\rightarrow$ 4123456 $\rightarrow$ 6412345 $\rightarrow$ 5641234 $\rightarrow$ 0564123 Destination.

(b) Describe a situation in which a linear network is better than a bus and another situation in which a bus is better than a linear network. Ignore cost. (7 pts)

Situations favorable to a bus: (1) Lots of broadcast messages. (Buses naturally broadcast, a linear network might have to send $N-1$ messages for each broadcast.) (2) Light traffic, but arbitrary source and destinations. (All processors are at the same, short, distance with a bus. In a linear network the average distance would be about $N/3$, very large.

Situation favorable to a linear network: heavy traffic but primarily between adjacent processors. The bus would be overloaded.

(c) The Moore bound is derived by constructing a tree. Does the diameter of that tree meet the Moore bound? Explain. (7 pts)

No. The distance from the root to a leaf is consistent with the Moore bound *but* the diameter of this tree is twice the distance from the root to a leaf. The tree is only a part of the arbitrary network being bounded, so there would be paths between leaves.

The paragraph below applies to the two following parts.

When applied to k-ary n-cubes, both the link-length bound and the fixed-bisection-width analysis suggest that smaller n is better.

(d) Explain how small-n KNC's are shown to be superior using each method. (Do not explain how the method works, but describe what happens in a large-n KNC that's bad [or a small-n KNC that's good] as revealed by each method.) (6 pts)

The link-length bound shows that links must be physically long in large-n networks, this contributes to latency. For sufficiently large N link-length effects will dominate latency.

Under the fixed bisection width analysis links are wider when n is smaller. Because of the small link width in large-n networks, messages must be broken into many packets, adding to latency.

(e) Are the two advantages of small-n KNC's discussed above orthogonal? (Roughly speaking, two things are orthogonal if they are different. For example, two orthogonal advantages of living in Louisiana are good food and good music. Two advantages of living in Louisiana that are not orthogonal are good music and good Jazz, since Jazz is a kind of music.) (3 pts)

Yes, if only one packet can be transmitted on a link at once (which is normally the case). In large-n networks, not only must messages travel along longer links, but those messages are broken into more pieces.