Name ______________________________

Interconnection Networks
EE 7725
Final Examination

9 December 1997, 10:00–12:00 CST

Problem 1	________	(36 pts)
Problem 2	________	(24 pts)
Problem 3	________	(12 pts)
Problem 4	________	(28 pts)
Exam Total	________	(100 pts)

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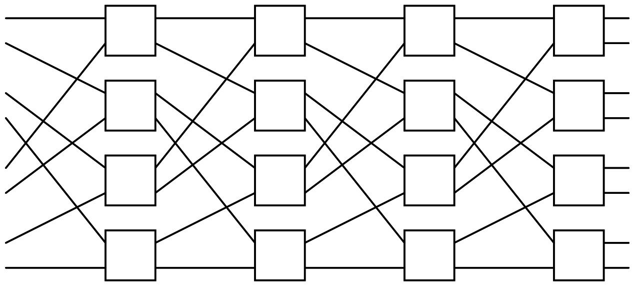
Good Luck!

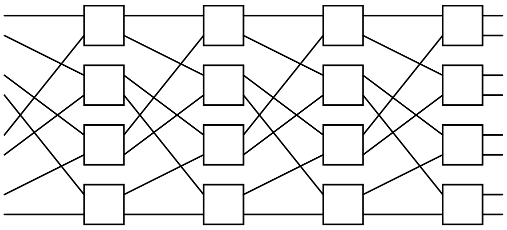
Problem 1: An $(m, n+1)$ extra-stage omega network is constructed from an (m, n) omega network by adding an extra stage without changing the number of inputs. (This is the same network used in homework 3/4). Let $\Omega_{m,n+1}$ denote the set of permutations admissible by an $(m, n+1)$ extra-stage omega.

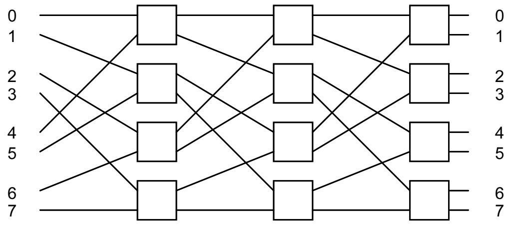
(*a*) Prove $|\Omega_{m,n+1}| < m!^{\left((n+1)m^{n-1}\right)}$, where $|\Omega_{m,n+1}|$ is the number of distinct permutations the network can admit. *Hint: There is a simple proof which uses the illustration.* (12 pts)



(*b*) Prove $\mathcal{I} \in \Omega_{m,n+1}$, where $\mathcal{I}$ is the identity permutation. *Hint: In the first stage request* $(s, d) \in \mathcal{I}$ *uses cell output* $\left(\sum_{i=0}^{n-1} s_{(i)}\right) \bmod m$. (12 pts)

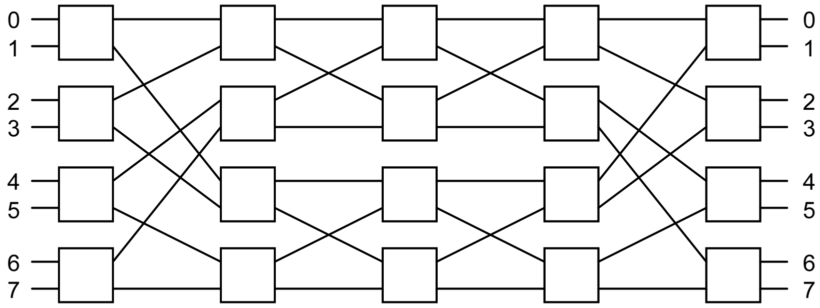
(c) Prove that there exists $P \in \Omega_{m,n+1}$ that is not in $\Omega_{m,n}$ for each $n > 2$. *Hints: Set all cells to identity. Consider a result from last year's final exam.* The illustrations below can be used to develop and explain your proof. (12 pts)





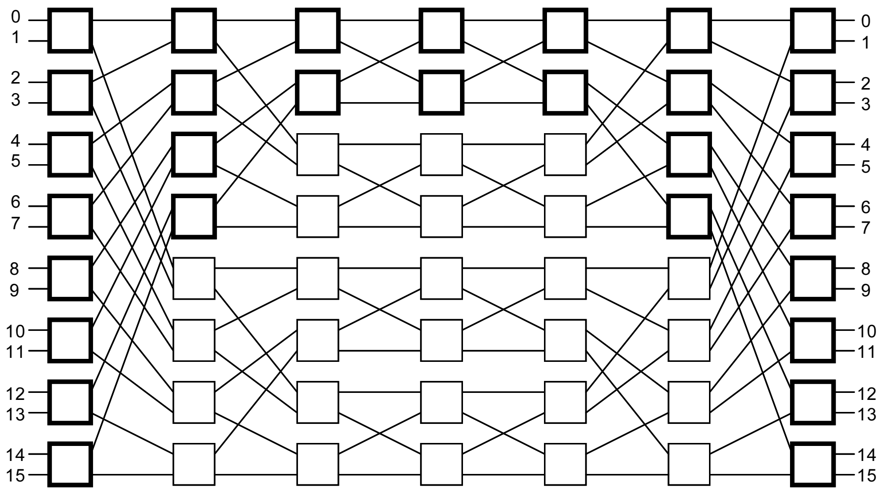
Problem 2: This problem contains three parts, each using Beneš networks (implicitly or explicitly). Full credit can be obtained by answering parts (*a*) and (*c*) only. Part (*b*) is for those who lack confidence in their solutions to (*a*) and (*c*). Credit will be given for all three parts, with a maximum of 24 points.

(*a*) Find a permutation for an 8-input Beneš network that can be routed in exactly eight ways. Justify your answer. (12 pts)

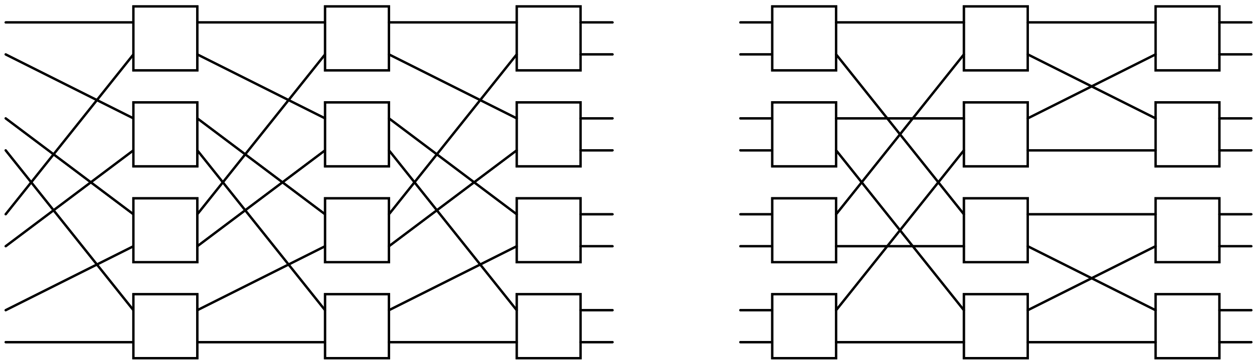


(*b*) (This part can be skipped if parts (*a*) and (*c*) are solved.) Route the following permutation for the bold cells only in the following 16-input Beneš network. (9 pts).

$$\begin{pmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 \\ 6 & 2 & 13 & 3 & 10 & 12 & 15 & 11 & 7 & 8 & 9 & 14 & 1 & 0 & 5 & 4 \end{pmatrix}$$

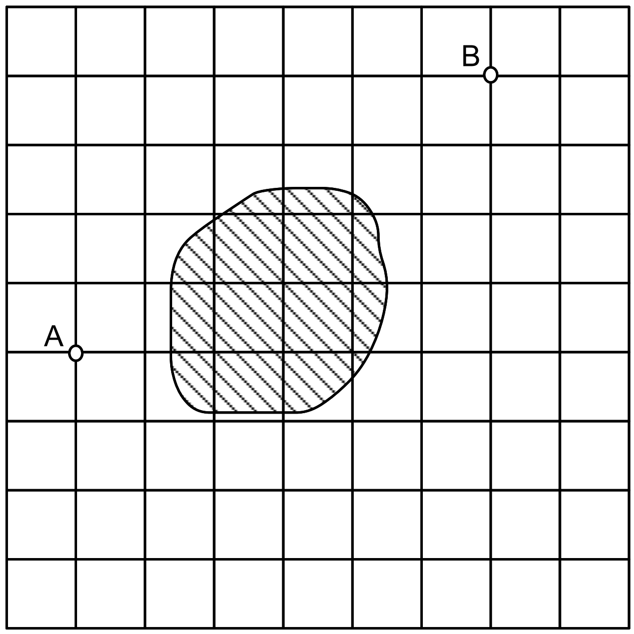


(c) Find a link pattern that can be placed between an (m, n) omega network and an (m, n) butterfly network (as illustrated below with (2,3) networks) so that the result is an m^n-input permutation network. *Hints: See the introduction to this problem. The link pattern is a digit-permute permutation.* (12 pts)



Problem 3: In the network below paths are to avoid the shaded region, **if possible**. There are no wrap-around links.

(a) Find a path from A to B using north-last routing. Find a path from A to B using east-first routing. (6 pts)



(b) Suppose the ascending/descending virtual channels added to a ring (k-ary 1-cube) were added along each dimension of a 2-dimensional torus (such as the one above with wraparound links added). Explain why this would not prevent deadlock. Give an example. (6 pts)

Problem 4: Answer each question below. Be brief, but not too brief.

(*a*) Draw an 8-input sorting network based on omega networks realizing bitonic permutations that uses 2×2 compare/exchange elements. (10 pts)

(*b*) Show the location of packets versus time of a five-packet message as it moves from node 0 to 3 in a linear (1-dimensional) network in an otherwise empty network using wormhole routing and a network using store-and-forward flow control (two situations). Show only the first nine cycles (assuming, as usual, one cycle for a packet to move from node to node). (9 pts)

(c) Show how the connection assignment below is routed in the pack and copy parts (but not the permute part) of the illustrated generalized connection network. (9 pts)

$$\begin{pmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 0,2,5 & 1,3 & & & 7 & 4,6 & & \end{pmatrix}$$