EE 7725 Homework 2 Due: 26 September 1997

A possible disadvantage of *low-dimension* k-ary n-cubes (those with small n, *e.g.* 1-dimensional) is their large diameter and average distance. A network with lower diameter, called the *crumpled cube*[1] is constructed from k-ary n-cubes with even $k \geq 4$ by adding links between all pairs of nodes that are in the same dimension and are distance $k/2$ apart. Odd and $k = 2$ crumpled cubes are not defined.

Problem 1: Define a crumpled cube as a graph. ($V = \langle k^n \rangle$, $E = \{\ldots\}$.)

$$V = \langle k^n \rangle$$

$$\begin{aligned} E =& \{\,(u, u + k^i) \mid u \in V,\, i \in \langle n \rangle\,,\, u_{(i)} < k-1\,\} \cup \\ & \{\,(u, u - (k-1)k^i) \mid u \in V,\, i \in \langle n \rangle\,,\, u_{(i)} = k-1\,\} \cup \\ & \{\,(u, u + \frac{k}{2}k^i) \mid u \in V,\, i \in \langle n \rangle\,,\, u_{(i)} < \frac{k}{2}\,\} \end{aligned}$$

Problem 2: What are the degree, diameter, and distance function ($d_{u,v}$) of a crumpled cube?

Degree: $\delta = 3n$

The distance is found by finding the cases where one of the new links should be taken. Distance:

$$d_{u,v} = \sum_{i=0}^{n-1} d'_{u_{(i)},v_{(i)}}$$

$$d'_{u_{(i)},v_{(i)}} = \begin{cases} 1 + d'_{u_{(i)},v_{(i)}+\frac{k}{2} \bmod k}, & \text{if } |u_{(i)} - v_{(i)}| > \frac{k+2}{4}; \\ |u_{(i)} - v_{(i)}|, & \text{otherwise.} \end{cases}$$

Diameter: $D = n\left\lfloor \frac{1}{4}(k+2) \right\rfloor$.

Problem 3: Which is better, a 2-D crumpled cube or some member of the k-ary n-cube family? State the criteria for, and justify, your choice.

Compare diameter of equal-cost networks having same number of nodes. Let cost be proportional to degree. Then a $n = 3$ KNC would have the same degree and cost as a crumpled cube. Since the dimension (n's) of the two networks are different, their radices (k's) will be written in terms of N and n: $k = N^{1/n}$. The diameter of the 3-D KNC is $3\lfloor N^{1/3}/2 \rfloor$ while the diameter of the crumpled cube is $2\lfloor (N^{1/2}+2)/4 \rfloor$. For large N, the KNC has lower diameter, and so is the better network.

Note: stating that for some n, the crumpled cube is more expensive but has lower diameter is a weaker comparison, and did not receive full credit.

[1] The name was made up for this problem. Crumpled cubes are roughly similar to express cubes (Dally, *IEEE Transactions on Computers*, vol. 40, no. 9, pp. 1016–1023, September 1991.) For maximum pedagogical effectiveness, solve the problem, then look up the reference.