Abstract Network Representations

Three commonly used:

- Link-graph model.
- Crosspoint-graph model.
- Sets-of-permutations model.

Uses of Abstract Representations

- Precisely describing network.
- Determining network properties.

  Isomorphism. (Similarity of two networks.)

Link-Graph Model (LGM)

Similar to direct-network graphs.

Network specified by four-tuple: (I, O, V, E) where:

V is a set of nodes, (each representing a cell, input, or output);

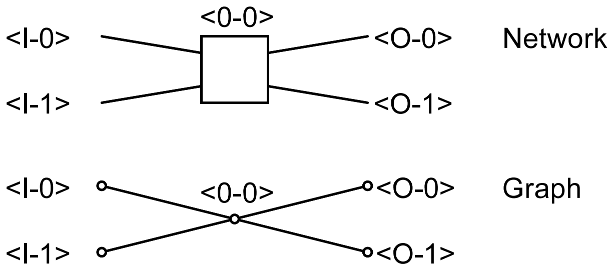
$I \subseteq V$ is a set of distinguished nodes, called inputs;

$O \subseteq V$ is a set of distinguished nodes, called outputs and;

$E \subseteq V \times V$ is a set of directed edges, (each representing a link).

The order in which links connect to cell *is not modeled.*

Link-Graph Model Simple Example



$$I = \{\langle \mathrm{I}, 0\rangle, \langle \mathrm{I}, 1\rangle\}$$

$$O = \{\langle \mathrm{O}, 0\rangle, \langle \mathrm{O}, 1\rangle\}$$

$$V = \{\langle \mathrm{I}, 0\rangle, \langle \mathrm{I}, 1\rangle, \langle \mathrm{O}, 0\rangle, \langle \mathrm{O}, 1\rangle, \langle 0, 0\rangle\}$$

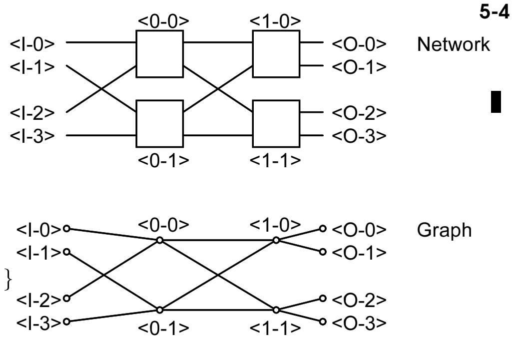
$$E = \{(\langle \mathrm{I}, 0\rangle, \langle 0, 0\rangle),\ (\langle \mathrm{I}, 1\rangle, \langle 0, 0\rangle),\ (\langle 0, 0\rangle, \langle \mathrm{O}, 0\rangle),\ (\langle 0, 0\rangle, \langle \mathrm{O}, 1\rangle)\}$$

MIN LGM Example

A $(2, 2)$ omega network:

Node notation:

$\langle$stage no. or I or O, cell no.$\rangle$.



$I = \{\langle \mathrm{I}, 0\rangle, \langle \mathrm{I}, 1\rangle, \langle \mathrm{I}, 2\rangle, \langle \mathrm{I}, 3\rangle\}$

$O = \{\langle \mathrm{O}, 0\rangle, \langle \mathrm{O}, 1\rangle, \langle \mathrm{O}, 2\rangle, \langle \mathrm{O}, 3\rangle\}$

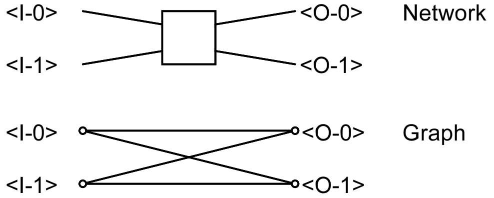
Stage-zero cell labels:

$\{\langle 0, 0\rangle, \langle 0, 1\rangle\}$.

$$V = \{\ \langle \mathrm{I}, 0\rangle, \langle \mathrm{I}, 1\rangle, \langle \mathrm{I}, 2\rangle, \langle \mathrm{I}, 3\rangle, \langle \mathrm{O}, 0\rangle, \langle \mathrm{O}, 1\rangle,$$
$$\langle \mathrm{O}, 2\rangle, \langle \mathrm{O}, 3\rangle, \langle 0, 0\rangle, \langle 0, 1\rangle, \langle 1, 0\rangle, \langle 1, 1\rangle\}.$$

$$E = \{(\langle \mathrm{I}, 0\rangle, \langle 0, 0\rangle),\ (\langle \mathrm{I}, 1\rangle, \langle 0, 1\rangle),\ (\langle \mathrm{I}, 2\rangle, \langle 0, 0\rangle),\ (\langle \mathrm{I}, 3\rangle, \langle 0, 1\rangle),$$
$$(\langle 0, 0\rangle, \langle 1, 0\rangle),\ (\langle 0, 0\rangle, \langle 1, 1\rangle),\ (\langle 0, 1\rangle, \langle 1, 0\rangle),\ (\langle 0, 1\rangle, \langle 1, 1\rangle),$$
$$(\langle 1, 0\rangle, \langle \mathrm{O}, 0\rangle),\ (\langle 1, 0\rangle, \langle \mathrm{O}, 1\rangle),\ (\langle 1, 1\rangle \langle \mathrm{O}, 2\rangle),\ (\langle 1, 1\rangle \langle \mathrm{O}, 3\rangle)\}.$$

CGM Simple Example



$$I = \{\langle \mathrm{I},\,0\rangle\,,\langle \mathrm{I},\,1\rangle\}$$

$$O = \{\langle \mathrm{O},\,0\rangle\,,\langle \mathrm{O},\,1\rangle\}$$

$$V = \{\langle \mathrm{I},\,0\rangle\,,\langle \mathrm{I},\,1\rangle\,,\langle \mathrm{O},\,0\rangle\,,\langle \mathrm{O},\,1\rangle\}$$

$$E = \{(\langle \mathrm{I},\,0\rangle\,,\langle \mathrm{O},\,0\rangle),\;(\langle \mathrm{I},\,0\rangle\,,\langle \mathrm{O},\,1\rangle),\;(\langle \mathrm{I},\,1\rangle\,,\langle \mathrm{O},\,0\rangle),\;(\langle \mathrm{I},\,1\rangle\,,\langle \mathrm{O},\,1\rangle)\}$$

LGM of an (m, n) Omega Network

Let $k = m^{n-1}$, as usual.

$I = \{\,\langle \mathrm{I},\,i\rangle \mid 0 \le i < mk\,\}$

$O = \{\,\langle \mathrm{O},\,i\rangle \mid 0 \le i < mk\,\}$

$V = I \cup O \cup \{\,\langle x,\,i\rangle \mid 0 \le x < n,\; 0 \le i < k\,\}$

The edges are the interesting part:

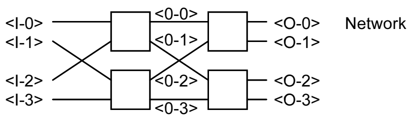
$$E = \{\,(\langle \mathrm{I},\,i\rangle\,,\langle 0,\,i \bmod k\rangle) \mid 0 \le i < mk\}\cup$$

$$\{\,(\langle x,\,i\rangle\,,\langle x+1,\,mi+j \bmod k\rangle) \mid$$

$$0 \le j < m,\; 0 \le i < k,\; 0 \le x < n-1\,\}\cup$$

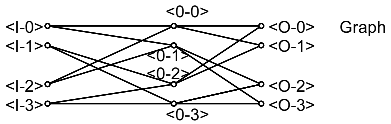
$$\{\,(\langle n-1,\,i\rangle\,,\langle \mathrm{O},\,mi+j\rangle) \mid 0 \le j < m,\; 0 \le i < k\,\}$$

Small Omega CGM Example



Network links $\rightarrow$ graph nodes:

$V = \{\;\langle \mathrm{I},\,0\rangle\,,\langle \mathrm{I},\,1\rangle\,,\langle \mathrm{I},\,2\rangle\,,\langle \mathrm{I},\,3\rangle\,,$
$\langle 0,\,0\rangle\,,\langle 0,\,1\rangle\,,\langle 0,\,2\rangle\,,\langle 0,\,3\rangle\,,$
$\langle \mathrm{O},\,0\rangle\,,\langle \mathrm{O},\,1\rangle\,,\langle \mathrm{O},\,2\rangle\,,\langle \mathrm{O},\,3\rangle\;\}.$



Stage-0, cell-0 crosspoints $\rightarrow$ graph edges:

$\{\,(\langle \mathrm{I},\,0\rangle\,,\langle 0,\,0\rangle),\;(\langle \mathrm{I},\,0\rangle\,,\langle 0,\,1\rangle),$
$(\langle \mathrm{I},\,2\rangle\,,\langle 0,\,0\rangle),\;(\langle \mathrm{I},\,2\rangle\,,\langle 0,\,1\rangle)\,\}$

All crosspoints $\rightarrow$ graph edges:

$E = \{\,(\langle \mathrm{I},\,0\rangle\,,\langle 0,\,0\rangle),\;(\langle \mathrm{I},\,0\rangle\,,\langle 0,\,1\rangle),\;(\langle \mathrm{I},\,2\rangle\,,\langle 0,\,0\rangle),\;(\langle \mathrm{I},\,2\rangle\,,\langle 0,\,1\rangle),$
$(\langle \mathrm{I},\,1\rangle\,,\langle 0,\,2\rangle),\;(\langle \mathrm{I},\,1\rangle\,,\langle 0,\,3\rangle),\;(\langle \mathrm{I},\,3\rangle\,,\langle 0,\,2\rangle),\;(\langle \mathrm{I},\,3\rangle\,,\langle 0,\,3\rangle),$
$(\langle 0,\,0\rangle\,,\langle \mathrm{O},\,0\rangle),\;(\langle 0,\,0\rangle\,,\langle \mathrm{O},\,1\rangle),\;(\langle 0,\,1\rangle\,,\langle \mathrm{O},\,2\rangle),\;(\langle 0,\,1\rangle\,,\langle \mathrm{O},\,3\rangle),$
$(\langle 0,\,2\rangle\,,\langle \mathrm{O},\,0\rangle),\;(\langle 0,\,2\rangle\,,\langle \mathrm{O},\,1\rangle),\;(\langle 0,\,3\rangle\,,\langle \mathrm{O},\,2\rangle),\;(\langle 0,\,3\rangle\,,\langle \mathrm{O},\,3\rangle)\,\}$

Crosspoint-Graph Model (CGM)

Not similar to direct-network graphs.

Network specified by four-tuple: (I, O, V, E) where:

- V is a set of nodes, (each representing a *link*);
- $I \subseteq V$ is a set of distinguished nodes, called inputs;
- $O \subseteq V$ is a set of distinguished nodes, called outputs and;
- $E \subseteq V \times V$ is a set of directed edges, (each representing a **crosspoint**).

Networks Using Incomplete Crossbars

These can be represented using the CGM but cannot be represented using the LGM.

A crossbar is *incomplete* if there is at least one input/output pair that cannot be connected.

Miscellaneous Topic: Connection Assignments

Important network characteristic:

Which sets of requests can be simultaneously routed?

Connection Assignment (CA)

A set of requests, for example, $\{ (1,2), (2,4) \}$.

Typically, network must simultaneously handle every request in a CA.

Permutation Connection Assignment
A connection assignment in which each input and each output appear exactly once.

For example, for a 3 input network:

$\{ (0,1),(2,0),(1,2) \}$ is a permutation CA.

$\{ (0,1),(0,0),(1,2) \}$ is not a permutation CA.
(Because input 0 connects to two outputs and input 2 does not appear).

A network *satisfies* a connection assignment . . .
. . . if there are non-conflicting paths for all requests.

Non-conflicting usually means link-disjoint.

If a network can satisfy a CA then the communication can be handled most efficiently.

If a network cannot satisfy a CA then:

In packet networks, there will be delay as some messages must wait in queues.

In circuit switched networks, the CA must be broken into two or more CAs . . .
. . . which then must be satisfied one at a time.

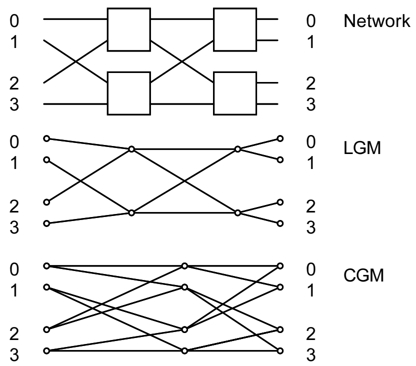
Graph Models and Connection Assignments

Represented Using LGM Models

Can satisfy a CA if there exist edge-disjoint paths for all requests.

Represented Using CGM Models

Can satisfy a CA if there exist node-disjoint paths for all requests.



Sets of Permutations Model

Set of all possible permutation connection assignments that the network can satisfy.

Covered in detail soon.

Comparison of Models

LGM might seem more natural.

CGM more powerful.

Set-of-permutations model is useful for studying what networks can do.