Name ______________________

Interconnection Networks
EE 7725
Midterm Examination

27 October 1997, 16:00–16:50+ C. S. T.

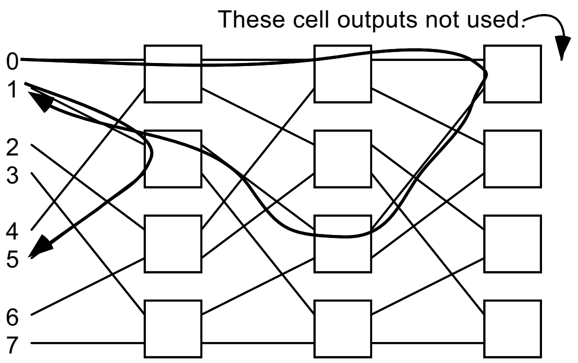
Modified

Problem 1	______	(40 pts)
Problem 2	______	(30 pts)
Problem 3	______	(30 pts)
Exam Total	______	(100 pts)

Alias Solution

Good Luck!

Problem 1: An (m, n) two-way omega network is constructed from an (m, n) omega network by eliminating outputs, adding two-way capability to all links, and allowing cells to connect their outputs (or what were their outputs in ordinary omega network cells) to their inputs and to connect any of their inputs to any of their outputs or inputs (reversing direction). What were inputs in an ordinary omega serve as both inputs and outputs in the two-way omega. Two paths in such a network are illustrated to the right:



(a) What is the cost of an (m, n) two-way omega network in crosspoints (using crosspoints that can pass a signal in one direction only)? (10 pts)

Divide crosspoints in a cell before the last stage into three groups: inputs to outputs (which are present in ordinary omegas), inputs to inputs, and outputs to inputs. The first group has m^2 crosspoints; the second, $m(m-1)$ (assuming a message cannot enter and immediately leave a cell by the same link); the third, m^2. Cells in the last stage do not have outputs, so they need only $m(m-1)$ crosspoints. The total is then: $\left(m^2 + m(m-1) + m^2\right) m^{n-1}(n-1) + m^2 m^{n-1}$.

(b) Find the cells on a shortest path from 12316_7 to 16416_7 in a (7,5) two-way omega network. Be sure to indicate the stage numbers. (10 pts)

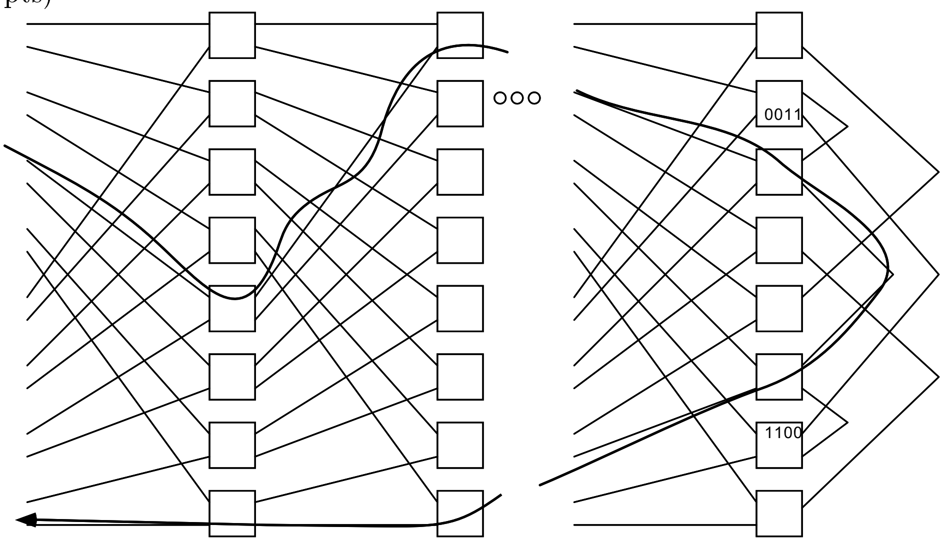
Position	Link	Cell
Input	$(12316)_7$	None
0 In	$(23161)_7$	$(2316)_7$
0 Out	$(2316p_0)_7$	$(2316)_7$
1 In	$(316p_02)_7$	$(316p_0)_7$
1 Out	$(316p_0p_1)_7$	$(316p_0)_7$
2 In	$(16p_0p_13)_7$	$(16p_0p_1)_7$
Back Towards Inputs ↩↩↩↩↩		
2 In	$(16p_0p_14)_7$	$(16p_0p_1)_7$
1 Out	$(416p_0p_1)_7$	$(416p_0)_7$
1 In	$(416p_06)_7$	$(416p_0)_7$
0 Out	$(6416p_0)_7$	$(6416)_7$
0 In	$(64161)_7$	$(6416)_7$
Input	$(16416)_7$	None

(c) Assuming a path can change direction exactly once (as the ones illustrated above do), how many paths are there between 12316_7 and 16416_7? (10 pts)

For the path illustrated above, p_0 and p_1 can be chosen 7^2 ways. A path between the terminals can go as far as stage 3 and 4; by similar reasoning there are 7^3 paths that turn at stage 3 and 7^4 paths that turn at stage 4, for a total of 2739 paths.

Problem 1, continued.

(d) Suppose the cell outputs in the last stage of an (m, n) two-way omega were connected to each other by a digit reverse permutation (ρ), as illustrated below. What would be the minimum number of stages needed so that there is at least one path between every pair of terminals? Justify your answer. (10 pts)



One way to solve the problem is to assume the network has x stages, find a path through the network, then determine what x should be. Consider such a path from s, with p_i the stage-i cell output used in the forward direction and p'_i the stage-i cell input used in the reverse direction:

Input	$s_{(n-1)} s_{(n-2)} \cdots s_{(0)}$
0 Out	$s_{(n-2)} s_{(n-3)} \cdots s_{(0)} p_0$
1 Out	$s_{(n-3)} s_{(n-4)} \cdots s_{(0)} p_0 p_1$
$\vdots$	$\vdots$
$x-1$ Out	$s_{(n-1-x)} s_{(n-2-x)} \cdots s_{(0)} p_0 \cdots p_{x-2} p_{x-1}$
Through Digit Reversal, Back Towards Inputs $\hookleftarrow\hookleftarrow\hookleftarrow\hookleftarrow\hookleftarrow$	
$x-1$ Out*	$p_{x-1} p_{x-2} \cdots p_0 s_{(0)} \cdots s_{(n-2-x)} s_{(n-1-x)}$
$x-1$ In	$p_{x-1} p_{x-2} \cdots p_0 s_{(0)} \cdots s_{(n-2-x)} p'_{x-1}$
$x-2$ In	$p'_{x-1} p_{x-1} p_{x-2} \cdots p_0 s_{(0)} \cdots s_{(n-3-x)} p'_{x-2}$
$x-3$ In	$p'_{x-2} p'_{x-1} p_{x-1} \cdots p_0 s_{(0)} \cdots s_{(n-4-x)} p'_{x-3}$
$\vdots$	$\vdots$
1 In	$p'_2 p'_3 p'_4 \cdots p'_{x-1} p_{x-1} \cdots p_0 s_{(0)} p'_1$
0 In	$p'_1 p'_2 p'_3 \cdots p'_{x-1} p_{x-1} \cdots p_0 p'_0$
Input	$p'_0 p'_1 p'_2 p'_3 \cdots p'_{x-1} p_{x-1} \cdots p_0$

The routing is the same as in a two-way omega, except for the digit reversal at stage $x-1$. The number of stages was made just large enough to remove all of s's digits. Since half of the n digits are removed in the forward path and the other half in the reverse, there must be $x = \lceil n/2 \rceil$ stages. Note that if n is odd, there will be multiple paths that use the last stage.

Problem 2: The *degenerate cube* is constructed from a k-ary n-cube by adding links between every pair of nodes that lie along a dimension that are not already connected. (Not just those at distance $k/2$, as in a crumpled cube.)

(a) Provide a graph definition of the degenerate cube. (8 pts)

Vertices: $V = \langle k^n \rangle$

Edges: $E = \{\, (u, u + dk^i) \mid u \in V,\ i \in \langle n \rangle\,,\ d \in \langle k \rangle\,,\ d + u_{(i)} < k - 1 \,\}$.

(b) Find expressions for the degree and diameter of the degenerate cube. (8 pts)

Degree: $\delta = (k-1)n$.

Diameter: $D = n$.

(c) Find an expression for the average distance in the degenerate cube. The expression need not be simplified. (7 pts)

Divide paths into sets based on how many radix-k digits must change from source to destination. For example, for a 10-ary 5 degenerate cube, nodes 12345 and 12300 differ in two digits. If two nodes differ in exactly i digits they are distance i apart. From any given node, there are $(k-1)^i\binom{n}{i}$ nodes that differ in exactly i digits. The average distance is then:

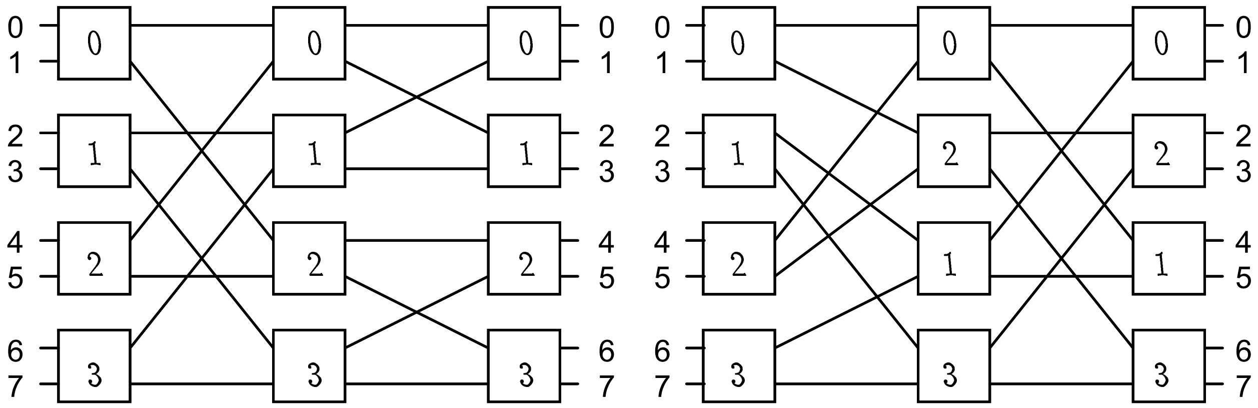
$$\overline{D} = \frac{1}{k^n - 1}\sum_{i=1}^{n}(k-1)^i\binom{n}{i} i = \frac{n(k^n - k)}{k^n - 1}$$

(d) Compare a degenerate cube with and without dimension-order routing to a de-Bruijn network with respect to the number of shortest paths for a request. Under dimension-order routing, which is better, taking into account the difference in cost? (7 pts)

Without dimension-order routing the degenerate cube has many more paths than the de-Bruijn when n is not small. With dimension-order routing of course, the degenerate cube can provide only one path. Since the cost of a degenerate cube is much higher, there is little advantage for it when dimension-order routing is used.

Problem 3: Answer each question below, be sure to properly justify each answer.

(a) Determine whether the two networks below are terminal equivalent and whether they are topologically equivalent. (8 pts)



They are not terminal equivalent, for one reason outputs 0 and 2 can reach the same center-stage cell in the left network but not in the right network. They are topologically equivalent; the cell labeling is shown above. The outputs of cells 1 and 2 in the last stage of the second network would also have to be re-labeled to match the outputs of cells 1 and 2 in the first network.

(b) As seen in class, deadlock in a N-ary 1-cube (the ring) can be avoided by dividing (most) links into two virtual channels, one for use by messages with lower destination numbers (than the current node) and the other for messages with higher destination numbers (remember?). Suppose in a de-Bruijn network links were one way (messages could use shuffle exchange links but not exchange inverse shuffle links), each link was divided into two virtual channels, one ascending and one descending, and messages were restricted to one of these channels based on their destinations. Is such a routing scheme deadlock free? If not, show an example of deadlock. (8 pts)

Use of virtual channels avoids deadlock in the ring because the channel dependency graph is a line, not a ring, so there can be no cycles. The de-Bruijn network contains many ring-like paths, not just one as the ring network. (For example, two rings in a $(4,3)$ de-Bruijn are: $0122 \to 1220 \to 2201 \to 2012 \hookleftarrow$ and $0121 \to 1210 \to 2101 \to 1012 \hookleftarrow$.) Since most paths cross from one ring to another (for example from 0122 to 0121) there are dependencies between rings that must be taken into account. In addition to dependencies between rings, another problem is determining which messages can use ascending and descending links. The determination cannot be made using node labels (numbers) of the current and destination node since within a ring they increase and decrease, inducing cycles in the dependency graph.

As an example of deadlock, consider the paths for requests (102,010) and (201,020) in a (3,3) dB:

102 D 020 D 201 D 010

201 D 010 A 102 D 020

where "D" and "A" indicate descending and ascending links. These requests cover a cycle in the channel-dependency graph, and so they can deadlock.

(c) Does the Moore bound prove that a network with the de-Bruijn network's degree, diameter, and number of nodes exist?[1] Explain. (7 pts)

No, it only shows that networks cannot exist with slightly smaller diameter.

(d) Consider two systems having N processors running a total of P tasks as described by the multithreaded system model. The network in system A suffers occasional congestion, while the congestion level in the network in system B is steady throughout a run (for a given amount of traffic it's always congested or it's always not congested). The systems are otherwise identical. Suppose for a certain number of tasks both systems have the same utilization, and it's less than one, A suffers occasional congestion and B suffers moderate but steady congestion. For which system might increasing the number of tasks be more worthwhile? (7 pts)

In system A increasing the number of tasks may increase utilization since the added traffic could be handled when the congestion clears. In B the network would never have a chance to catch up.

[1] Just because a question is asked during review doesn't mean that it won't appear on the exam. For the record, the exam was made up before the review.