Previously-Assigned Homework

The problems below were assigned in previous semesters. Only those problems relevant to the Fall 1995 semester are included. Note that the solution to some problems were presented in class in the Fall 1995 semester, but were not presented when the problems were assigned. (So the problems look easier than they were.) Solutions to a few of the problems are available. See the previous-work Web page.

EE 7725 Homework 1 Due: 30 September 1994

Problem 1: Compare a k-ary n-cube and a deBruijn network by plotting diameter v. degree (δ) with N fixed.

Problem 2: For each node, 0, 1, and 5, list all nodes at distances 1, 2, 3, and 4 (if any) in a deBruijn network with $r = 2$ and $n = 4$.

The solution should look something like this: Node 0, nodes at distance 1: 1,8; nodes at distance 2: 2,...

EE 7725 Homework 2 Due: 14 October 1994

Problem 1: Find the throughput and service probability for an input of a single, unbuffered crossbar with a inputs and b outputs.

EE 7725 Homework 3 Due: 26 October 1994

Problem 1: Consider two requests for an $(n, 2, 2^{n-1}, \mathrm{S}, \mathrm{T})$ omega network: request $A = (a, \alpha)$ and request $B = (b, \beta)$.

It is impossible to route certain pairs of requests in this network. Describe those pairs. (The answer should look something like $b_{(0)} = \sqrt{2\pi(a_{(0)} + a^2_{(x)})}$ or $b_{(1)} = \ldots$)

How could those pairs be routed in network $(n+1, 2, 2^{n-1}, \mathrm{S}, \mathrm{T})$?

Show three requests, $A = (a, \alpha)$, request $B = (b, \beta)$, and request $C = (c, \gamma)$ which could not be simultaneously routed in this network.

EE 7725 Homework 4 Due: 7 November 1994

Problem 1: Consider a system using an $(n+1, m, m^{n-1}, \mathrm{T}, \mathrm{T})$ extra-stage cube network. The system has hot spots, the ESC is used to minimize the effects of the resulting saturation trees in the following way. Messages bound for the hot spot would always be routed the same way. Messages not bound for the hot spot are routed on one of the paths (randomly chosen) that does not pass through the saturation tree, if any. Suppose there is a hot spot at output H which forms a saturation tree reaching stage x. Show which stage-0 cell outputs request (a, α) could use.

Problem 2: A banyan network with combining cells is to be used to implement a barrier that will work in the following way. A PE enters a barrier by sending a “barrier in” message. The PE will then wait for a “barrier out” message to arrive, after which it will resume computation. A PE will not receive a “barrier out” until all PEs have sent “barrier in” messages. Show how a combining network (or something similar to a combining network) can be used to implement this. There should be no bottlenecks in the network; that is each cell should only have to handle $O(1)$ messages (in a cycle) regardless of when PEs send “barrier in” messages.

Assume PEs will send only one “barrier in” message.

EE 7725 Homework 5 Due: 11 November 1994

Problem 1: A fault tolerant network adds intra-stage (auxiliary) links to the last stage of an omega network. The other stages are unchanged. These links are placed between all adjacent cells and the cells 0 and $N/2-1$. A message can avoid a fault by modifying its destination; the destination is modified so that the fault will be avoided. When the message reaches the last stage it uses the intra-stage links (if necessary) to move to the cell it would have been in if its destination were not modified. It then exits the network.

(a) Explain how a fault in stage i link j can be avoided in this network. Your solution should have the message travel the minimum possible distance.

(b) Explain which single failures this network would not tolerate.

Problem 2: What is the CGM representation of a $(n, m, m^{n-1}, \mathrm{S}, \mathrm{T})$ MIN?

EE 7725 Homework 6 Due: 30 November 1994

Problem 1: Prove that a butterfly network is not terminal equivalent to an omega network.

Problem 2: Prove that a baseline network is terminal equivalent to an inverse baseline.

Problem 3: How is request (a, α) routed in an inverse butterfly network? Be sure to check your answer.

Problem 4: What is the LGM of a Clos network?

EE 7725 Homework 7 Due: Not collected.

Problem 1: Route a 16-input Beneš network for permutation connection assignment

$$P = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 \\ 4 & 0 & 7 & 12 & 15 & 3 & 1 & 9 & 2 & 6 & 13 & 8 & 14 & 11 & 10 & 5 \end{pmatrix}.$$

Problem 2: Show how the permutation above would be routed on a $(3, 4, 4, \mathrm{T}, \mathrm{T})$ Clos network.

Problem 3: Write a program to route a permutation in a $(3, 2^{\mu}, 2^{n-\mu}, \mathrm{T}, \mathrm{T})$ Clos network. C++ and pseudocode solutions might be placed at the URL given above.

Problem 4: An 2^n-input inverse omega network (having more than 4 inputs) is to be used by 4 users. The inputs of the inverse omega network are to be divided evenly among the users. The part of the network used by each user act just like a 2^{n-2}-input inverse omega network. No set of requests sent by one user can affect any other user. For example, the congestion caused by user A should not delay messages sent by users B, C, or D. Explain how inputs and outputs can be divided among users to achieve this. Explain how routing would be done. (See J-partitioning in Note Set 5 for the solution.)

Problem 5: Show how connection assignment

$$\{\,(1,0),\ (1,1)\,(1,6)\,(2,15)\,(4,4)\,(4,3)\,(13,2)\,(13,14)\,\}$$

would be routed on a pack/copy/permute generalized connector. In addition to the final cell settings, show the following: the packing CA, copy CA, and permute CA needed.

Problem 6: Prove that an inverse omega network followed by an omega network can realize all permutation connection assignments. *Hint: Consider the recursive formulation of these networks. Note the similarity of the combined networks to a Clos network.*

EE 7725 Homework 1 Due: 17 September 1993

Problem 1: A toroidal version of the $k-$ ary $n-$ cube can be defined. In such a network the links at the edges "wrap around" to the other side. (For example, a ring is a torodial version of the linear network. Define such a network (similar to the definition provided in note set one). What is the diameter and cost of this network?

Problem 2: Let the cost of a $k-$ary $n-$cube be $C = wnk^n$ *link slices* (w link slices make up one link) and the average distance for an algorithm of interest be $h = k^n/n$ links. Find the values of k and n which would give the highest utilization (U) for a fixed cost (C) and number of PEs (N). Your answer should be in terms of C, t_c, M, N, and t_p.

EE 7725 Homework 2 Due: 24 September 1993

Problem 1: Give the 5-tuple notation $(s, \vec{m}, \vec{k}, S, T)$ of the lowest cost generalized omega network and modified omega network having 2736 inputs. It is not necessary to draw the network.

Problem 2: Give the link and cell numbers along the path taken by request $(173, 2008)$ in a $(5, (3, 4, 5, 6, 7), -, S, T)$ generalized omega network. It is not necessary to draw the network.

EE 7725 Homework 4 Due: 29 October 1993

Problem 1: A network is to handle messages with a wide variation in message sizes. Discuss the tradoffs of using cells with statically allocated fully connected (SAFC), dynamically allocated bused (DAMQ), and statically allocated bused (SAMQ) buffers. (The items in parenthesis are the abbreviations used in the paper by Tamir and the glossary in Note Set 3.)

Problem 2: A fault tolerant network adds intra-stage (vertical) links to the last stage of an omega network. The other stages are unchanged. These links are placed between adjacent cells and the cells 0 and $N/2-1$. (If you're not sure what I mean, ask.) A message can avoid a fault by modifying its destination; the destination is modified so that the fault will be avoided. When the message reaches the last stage it uses the intra-stage links (if necessary) to move to the cell it would have been in if its destination were not modified. It then exits the network.

(a) Explain how a fault in stage i link j can be avoided in this network. Your solution should have the message travel the minimum possible distance.

(b) Explain which single failures this network would not tolerate.

EE 7725 Homework 4 Due: 12 November 1993

Problem 1: Show how the following permutation can be realized on a 16 input omega network using J-partitions:

$$\begin{pmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 \\ 1 & 4 & 15 & 14 & 5 & 0 & 11 & 10 & 9 & 8 & 6 & 7 & 13 & 12 & 2 & 3 \end{pmatrix}.$$

Give the set J and the permutations X and Y_i. Identify the family to which each permutation belongs. Hint: $|J| = 2$.

Problem 2: Find the functions (f_i) for the permutation

$$\begin{pmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 2 & 6 & 1 & 5 & 7 & 0 & 4 & 3 \end{pmatrix}.$$

Problem 3: Let $c \in \langle 2^n \rangle$. A permutation C_c is the c complement permutation if for all $x \in \langle 2^n \rangle$

$$xC_c = x \oplus c,$$

where $\oplus$ is the bitwise exclusive or operation. The complement family is the set of permutations $\mathcal{C}(2^n) = \{C_c | c \in \langle 2^n \rangle\}$. Can the complement family can be satisfied by an omega network? Prove your answer.

EE 7700-2 Homework I Due: 9 September 1992

Problem 1: A folded hypercube is a (2,n) cube with a set of links added between nodes a and $\overline{a}$.* What is the diameter of this network?

EE 7700-2 Homework II Due: 23 September 1992

Problem 1: Give the MIN representation for the least expensive generalized omega network of 60 inputs. Describe cost model used.

Problem 2: Find all possible routing tags for request (89,3) in a (5,3,30,S,T) modified omega network.

EE 7700-2 Homework III Due: 9 October 1992

Problem 1: Show that the following two networks are topologically equivalent:
Network A: Start with a $(2,3)$ cube. Replace each link with a 2×2 cell. Replace each node with three links. Each of the three links connects to a cell input and output such that there is a loop of length three if cells are set to identity.
Network B: Start with a $(3, 2, 4, S, T)$ flat regular MIN. Connect input i to output i for $i \in \langle 8 \rangle$.

Problem 2: Give the CGM of a $(2, 3, -, S, T)$. Draw the graph and show the sets I, O, and V. Indicate how many edges are in E and show five of them.

EE 7700-3 Homework IV Due: 9 November 1992

Problem 1: Design two types of cells that could be used to implement a k-ary n-cube operated in circuit switched mode. Each node is to have one cell. The cell should have one terminal for each link; the links are to be bidirectional. The cell should also have a connection to the PE associated with the node. The cell should be able to make the proper connections between the terminals. Do not show control logic. In one design, at most one path can pass through a node. In the other design, many signals can pass through a node, but each link is limited to one signal. Give the cost of each cell design.

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* The notation $\overline{a}$ indicates the number obtained by inverting each bit in a's binary representation (the number of digits is understood). E.g., $\overline{12} = 3$ (4 digits) and $\overline{17} = 110$ (7 digits).

Problem 2: Express Ω_N in terms of permutations and sets of permutations based upon the recursive version of the omega network. (Hint: see the definitions of upper and lower triangular permutations in Note Set #4.)

EE 7700-3 Homework V Due: 16 November 1992

Problem 1: Find the functions (f_i) for the permutation

$$\begin{pmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 3 & 2 & 0 & 7 & 5 & 4 & 6 & 1 \end{pmatrix}.$$

Problem 2: Let $c \in \langle 2^n \rangle$. A permutation C_c is the c complement permutation if for all $x \in \langle 2^n \rangle$

$$xC_c = x \oplus c,$$

where $\oplus$ is the bitwise exclusive or operation. The complement family is the set of permutations $\mathcal{C}(2^n) = \{C_c | c \in \langle 2^n \rangle\}$. Can the complement family can be satisfied by an omega network? Prove your answer.

EE 7700-3 Homework VI Due: 23 November 1992

Problem 1: Modify the definition of J partitioning to include omega networks built using $m \times m$ cells. Assume that only an entire cell can be frozen. Include appropriately modified versions of the offset and block functions.

EE 7700-3 Homework VII Due: 2 December 1992

Problem 1: Use the looping algorithm to route the permutation

$$\begin{pmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 \\ 3 & 4 & 6 & 0 & 1 & 9 & 2 & 10 & 11 & 5 & 7 & 8 \end{pmatrix}$$

in an $m = 4, k = 3$ Clos network. Show the permutation for each of the cells in the Clos network and for the imaginary networks.

Problem 2: There is a generalized omega network of N inputs for each factoring of N. Define a similar generalized Beneš network. For a given N how does one obtain the minimum cost network? Give an example for $N = 30$.

EE 7700-3 Review Problems Due: Will not be collected.

Problem 1: Draw the 8-input sorting network based on Batcher's bitonic sort. Choose 8 random numbers (or the number of calories per serving of 8 of your favorite foods). Show how the sorting network sorts these numbers.

Problem 2: Consider an implementation of a parallel computer using a k-ary n-cube. Suppose the physical length of each link were bounded (for example, no longer than 1 meter). Also suppose the physical volume of each processor were bounded (for example no more than one liter). Explain why a parallel computer using a (k, n) with $n > 3$ is not scalable. Suppose the link length restriction were dropped. Would the processor then be scalable?

Problem 3: Suppose an omega network is to be implemented on integrated circuits with one cell per chip. Further, assume that cost is measured only in terms of pins. What is the best cell size for an N input network? Is this reasonable?

Problem 4: Is the inverse omega network a delta network?

Problem 5: Give the CGM of a $(2, 3)$ k-ary n-cube. Make a reasonable assumption about which nodes are inputs and outputs. Draw the graph and show the sets I, O, and V. Indicate how many edges are in E and show five of them.

Problem 6: Find the throughput of an unbuffered, packet dropping crossbar assuming only j inputs send messages. As in class, message destinations are uniformly distributed.

Problem 7: The simulations presented in class showed that shorter messages have lower latency than longer messages for a given traffic intensity. Does that necessarily mean that large messages should be broken into smaller ones? Explain.

Problem 8: A network is to handle messages with a wide variation in message sizes. Discuss the tradoffs of using cells with SAFC, DAMQ, and SAMQ buffers.

Problem 9: Are the shuffle permutations a subset of the shift permutations?

Problem 10: Is the mirror image of a permutation network also a permutation network? Is the mirror image of a generalized connector also a generalized connector?

Problem 11: Is the omega network based concentrator self routing? Explain.