Abstract Network Representations

Three commonly used:

- Link-graph model.
- Crosspoint-graph model.
- Sets-of-permutations model.

Uses of Abstract Representations

- Precisely describing network.
- Determining network properties.

  Isomorphism. (Similarity of two networks.)

Link-Graph Model (LGM)

Similar to direct-network graphs.

Network specified by four-tuple: (I, O, V, E) where:

V is a set of nodes, (each representing a cell, input, or output);

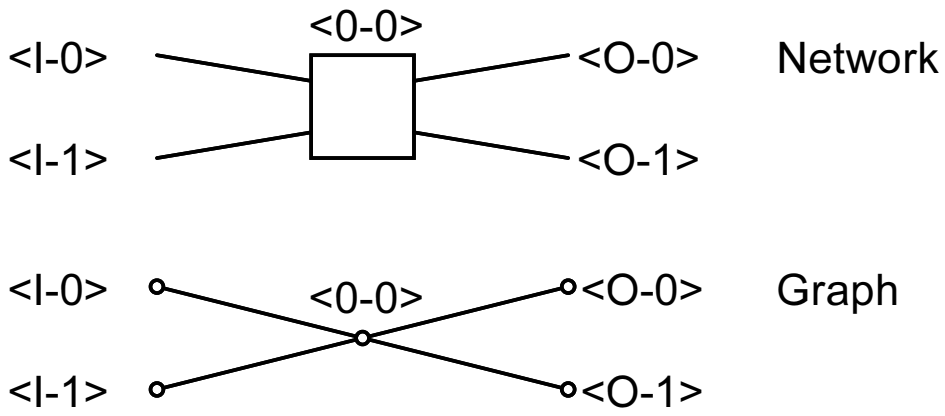
$I \subseteq V$ is a set of distinguished nodes, called inputs;

$O \subseteq V$ is a set of distinguished nodes, called outputs and;

$E \subseteq V \times V$ is a set of directed edges, (each representing a link).

The order in which links connect to cell *is not modeled*.

Simple Example



$$I = \{\langle \mathrm{I},\, 0\rangle, \langle \mathrm{I},\, 1\rangle\}$$

$$O = \{\langle \mathrm{O},\, 0\rangle, \langle \mathrm{O},\, 1\rangle\}$$

$$V = \{\langle \mathrm{I},\, 0\rangle, \langle \mathrm{I},\, 1\rangle, \langle \mathrm{O},\, 0\rangle, \langle \mathrm{O},\, 1\rangle, \langle 0,\, 0\rangle\}$$

$$E = \{(\langle \mathrm{I},\, 0\rangle, \langle 0,\, 0\rangle),\ (\langle \mathrm{I},\, 1\rangle, \langle 0,\, 0\rangle),\ (\langle 0,\, 0\rangle, \langle \mathrm{O},\, 0\rangle),\ (\langle 0,\, 0\rangle, \langle \mathrm{O},\, 1\rangle)\}$$

MIN LGM Example

$(2, 2, 2, \mathrm{S}, \mathrm{T})$ network:

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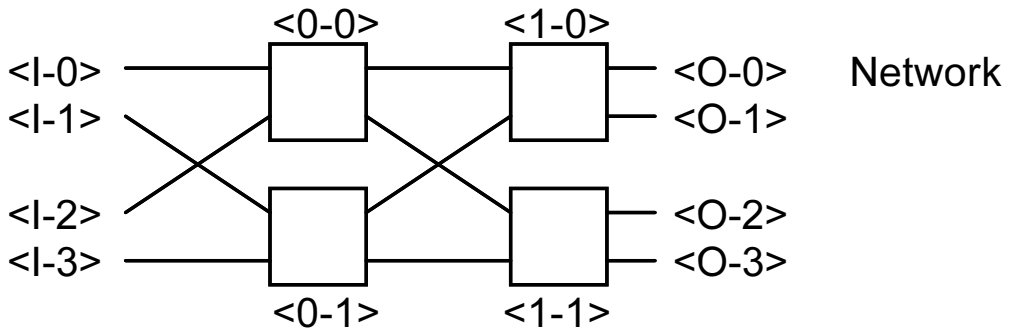
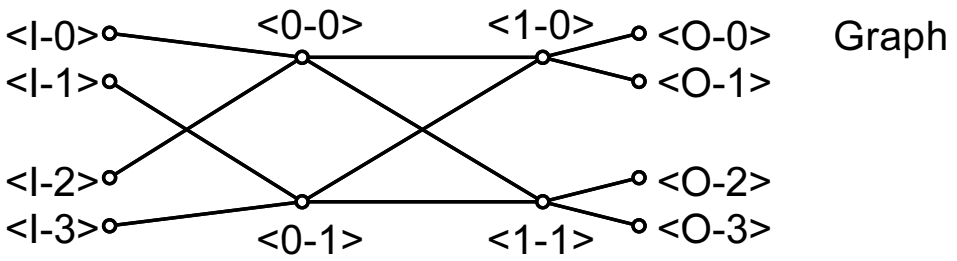




Node notation: $\langle$stage no. or I or O, cell no.$\rangle$.

$I = \{\langle \mathrm{I},\, 0\rangle, \langle \mathrm{I},\, 1\rangle, \langle \mathrm{I},\, 2\rangle, \langle \mathrm{I},\, 3\rangle\} \quad O = \{\langle \mathrm{O},\, 0\rangle, \langle \mathrm{O},\, 1\rangle, \langle \mathrm{O},\, 2\rangle, \langle \mathrm{O},\, 3\rangle\}$

Stage-zero cell labels: $\{\langle 0,\, 0\rangle, \langle 0,\, 1\rangle\}$.

$$V = \{\, \langle \mathrm{I},\, 0\rangle, \langle \mathrm{I},\, 1\rangle, \langle \mathrm{I},\, 2\rangle, \langle \mathrm{I},\, 3\rangle, \langle \mathrm{O},\, 0\rangle, \langle \mathrm{O},\, 1\rangle, \langle \mathrm{O},\, 2\rangle, \langle \mathrm{O},\, 3\rangle, \langle 0,\, 0\rangle, \langle 0,\, 1\rangle, \langle 1,\, 0\rangle, \langle 1,\, 1\rangle\}.$$

$$E = \{(\langle \mathrm{I},\, 0\rangle, \langle 0,\, 0\rangle),\ (\langle \mathrm{I},\, 1\rangle, \langle 0,\, 1\rangle),\ (\langle \mathrm{I},\, 2\rangle, \langle 0,\, 0\rangle),\ (\langle \mathrm{I},\, 3\rangle, \langle 0,\, 1\rangle),$$

$$(\langle 0,\, 0\rangle, \langle 1,\, 0\rangle),\ (\langle 0,\, 0\rangle, \langle 1,\, 1\rangle),\ (\langle 0,\, 1\rangle, \langle 1,\, 0\rangle),\ (\langle 0,\, 1\rangle, \langle 1,\, 1\rangle),$$

$$(\langle 1,\, 0\rangle, \langle \mathrm{O},\, 0\rangle),\ (\langle 1,\, 0\rangle, \langle \mathrm{O},\, 1\rangle),\ (\langle 1,\, 1\rangle \langle \mathrm{O},\, 2\rangle),\ (\langle 1,\, 1\rangle \langle \mathrm{O},\, 3\rangle)\}.$$

LGM of an $(n, m, m^{n-1}, \mathrm{S}, \mathrm{T})$ Omega Network

Let $k = m^{n-1}$, as usual.

$I = \{ \langle \mathrm{I},\, i \rangle \mid 0 \le i < mk \}$

$O = \{ \langle \mathrm{O},\, i \rangle \mid 0 \le i < mk \}$

$V = I \cup O \cup \{ \langle x,\, i \rangle \mid 0 \le x < n,\ 0 \le i < k \}$

The edges are the interesting part:

$$E = \{ (\langle \mathrm{I},\, i \rangle, \langle 0,\, i \bmod k \rangle) \mid 0 \le i < mk \} \cup$$

$$\{ (\langle x,\, i \rangle, \langle x+1,\, mi + j \bmod k \rangle) \mid$$

$$0 \le j < m,\ 0 \le i < k,\ 0 \le x < n-1 \} \cup$$

$$\{ (\langle n-1,\, i \rangle, \langle \mathrm{O},\, mi + j \rangle) \mid 0 \le j < m,\ 0 \le i < k \}$$

Crosspoint-Graph Model (CGM)

Not similar to direct-network graphs.

Network specified by four-tuple: (I, O, V, E) where:

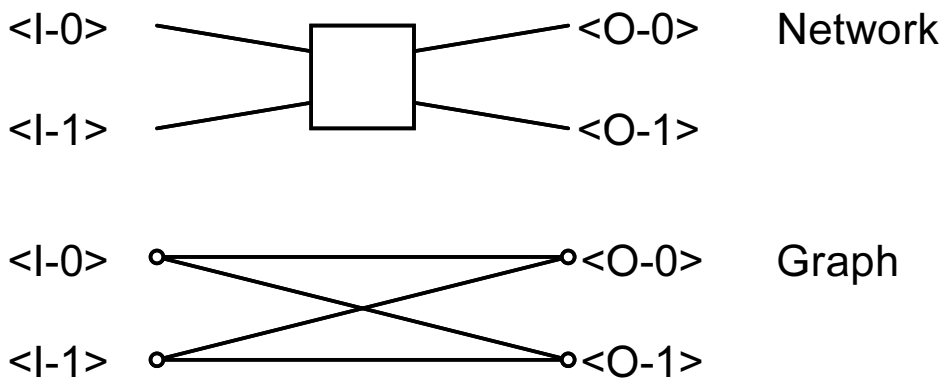
V is a set of nodes, (each representing a *link*);

$I \subseteq V$ is a set of distinguished nodes, called inputs;

$O \subseteq V$ is a set of distinguished nodes, called outputs and;

$E \subseteq V \times V$ is a set of directed edges, (each representing a **crosspoint**).

Simple Example



$$I = \{\langle \mathrm{I},\, 0\rangle, \langle \mathrm{I},\, 1\rangle\}$$

$$O = \{\langle \mathrm{O},\, 0\rangle, \langle \mathrm{O},\, 1\rangle\}$$

$$V = \{\langle \mathrm{I},\, 0\rangle, \langle \mathrm{I},\, 1\rangle, \langle \mathrm{O},\, 0\rangle, \langle \mathrm{O},\, 1\rangle\}$$

$$E = \{(\langle \mathrm{I},\, 0\rangle, \langle \mathrm{O},\, 0\rangle),\ (\langle \mathrm{I},\, 0\rangle, \langle \mathrm{O},\, 1\rangle),\ (\langle \mathrm{I},\, 1\rangle, \langle \mathrm{O},\, 0\rangle),\ (\langle \mathrm{I},\, 1\rangle, \langle \mathrm{O},\, 1\rangle)\}$$

$(2, 2, 2, \mathrm{S}, \mathrm{T})$ MIN CGM Example

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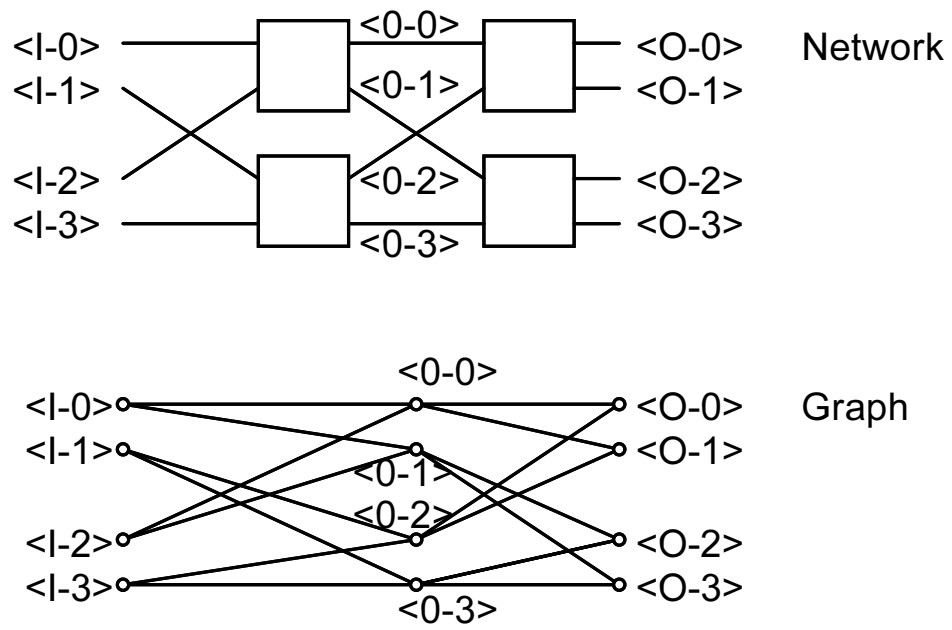

Network links $\rightarrow$ graph nodes:

$$V = \{ \langle \mathrm{I},\, 0\rangle, \langle \mathrm{I},\, 1\rangle, \langle \mathrm{I},\, 2\rangle, \langle \mathrm{I},\, 3\rangle, \langle 0,\, 0\rangle, \langle 0,\, 1\rangle, \langle 0,\, 2\rangle, \langle 0,\, 3\rangle, \langle \mathrm{O},\, 0\rangle, \langle \mathrm{O},\, 1\rangle, \langle \mathrm{O},\, 2\rangle, \langle \mathrm{O},\, 3\rangle \,.$$

Stage-0, cell-0 crosspoints $\rightarrow$ graph edges:

$$\{(\langle \mathrm{I},\, 0\rangle, \langle 0,\, 0\rangle),\ (\langle \mathrm{I},\, 0\rangle, \langle 0,\, 1\rangle),\ (\langle \mathrm{I},\, 2\rangle, \langle 0,\, 0\rangle),\ (\langle \mathrm{I},\, 2\rangle, \langle 0,\, 1\rangle)\}$$

All crosspoints $\rightarrow$ graph edges:

$$\begin{aligned} E = \{&(\langle \mathrm{I},\, 0\rangle, \langle 0,\, 0\rangle),\ (\langle \mathrm{I},\, 0\rangle, \langle 0,\, 1\rangle),\ (\langle \mathrm{I},\, 2\rangle, \langle 0,\, 0\rangle),\ (\langle \mathrm{I},\, 2\rangle, \langle 0,\, 1\rangle),\\ &(\langle \mathrm{I},\, 1\rangle, \langle 0,\, 2\rangle),\ (\langle \mathrm{I},\, 1\rangle, \langle 0,\, 3\rangle),\ (\langle \mathrm{I},\, 3\rangle, \langle 0,\, 2\rangle),\ (\langle \mathrm{I},\, 3\rangle, \langle 0,\, 3\rangle),\\ &(\langle 0,\, 0\rangle, \langle \mathrm{O},\, 0\rangle),\ (\langle 0,\, 0\rangle, \langle \mathrm{O},\, 1\rangle),\ (\langle 0,\, 1\rangle, \langle \mathrm{O},\, 2\rangle),\ (\langle 0,\, 1\rangle, \langle \mathrm{O},\, 3\rangle),\\ &(\langle 0,\, 2\rangle, \langle \mathrm{O},\, 0\rangle),\ (\langle 0,\, 2\rangle, \langle \mathrm{O},\, 1\rangle),\ (\langle 0,\, 3\rangle, \langle \mathrm{O},\, 2\rangle),\ (\langle 0,\, 3\rangle, \langle \mathrm{O},\, 3\rangle)\} \end{aligned}$$

Networks Using Incomplete Crossbars

These can be represented using the CGM but cannot be represented using the LGM.

A crossbar is *incomplete* if there is at least one input/output pair that cannot be connected.

Miscellaneous Topic: Connection Assignments

Important network characteristic: which sets of requests can be simultaneously routed?

Connection Assignment (CA)– A set of requests.

For example, $\{(1,2),(2,4)\}$.

Usually, one intends to simultaneously use all requests in a connection assignment.

Permutation Connection Assignment– A set of requests for a network in which each input and each output appear exactly once.

For example, for a 3 input network:

$\{(0,1),(2,0),(1,2)\}$ is a permutation CA.

$\{(0,1),(0,0),(1,2)\}$ is not a permutation CA (because input 0 connects to two outputs and input 2 does not appear).

A network *satisfies* a connection assignment if there are non-conflicting paths for all requests.

If a network can satisfy a CA then the communication can be handled most efficiently.

If a network cannot satisfy a CA then either:

There will be delay as some messages must wait in queues. (In the packet switching systems we have studied.)

The CA must be broken into two or more CAs, which then must be satisfied one at a time. (In the circuit switching systems we will soon be studying.)

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Graph Models and Connection Assignments

A network can satisfy a connection assignment if there exist edge-disjoint paths in the network's LGM for all requests in the connection assignment.

A network can satisfy a connection assignment if there exist node-disjoint paths in the network's CGM for all requests in the connection assignment.

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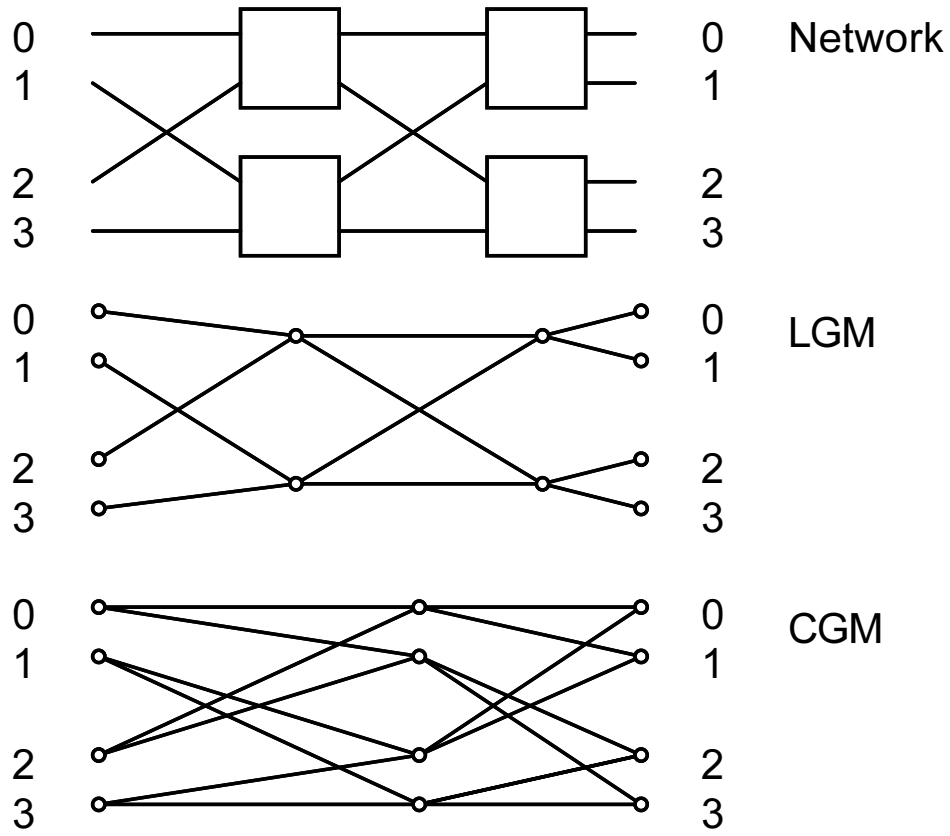

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Sets of Permutations Model

Set of all possible permutation connection assignments that the network can satisfy.

Covered in detail soon.

Comparison of Models

LGM might seem more natural.

CGM more powerful.

Set-of-permutations model is useful for studying what networks can do.