EE – 3410 Electric Power

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Electromechanical Energy Conversion

Introduction to Electric Machines

1. The very first experience with electric (linear) motors

An operation of any electromechanical device, in that number electric machines, it is electric motors and generators, can be seen as:

a) An impact of magnetic field on a piece of ferromagnetic material,
b) An interaction of two magnetic fields: magnetic field of primary part and magnetic field of secondary part; the secondary magnetic field can be generated:
   - either by a current flowing in a winding (conductor),
   - or by permanent magnet.

The machines that belong to the group (a) are called the *reluctance motors*, and the machines that operates on the basis of the mode (b) are the rest of the majority of electric machines, it is generators and motors. We will discuss these various electric machines starting with our very first experience with permanent magnets. In electric machines the basic requirement for the primary magnetic field is: it has to be in motion. It can move with:

- translation motion – we call it a magnetic travelling field, which exists in linear machines
- rotary motion – a rotating magnetic field, which is in rotating machines
- complex motion - described by two or three space co-ordinates, which is generated in electric motors with two degrees of mechanical freedom, e.g.:
  - rotary-linear motors with helical motion of the rotor,
  - motors with spherical rotor
  - X-Y linear motors.

In the following sections we will concentrate on the linear machines. The rotating machines will be seen as a geometrical conversion of the flat linear structure into the cylindrical or disc structure.

1.1. Reluctance (linear) synchronous motor

Imagine, we have the bar permanent magnet and we slide it under the table-top as shown in Fig.1. Above is the iron bar, which is pulled by the magnetic field of permanent magnet. In this perticular case the magnetic travelling field is obtained by the permanent magnet, called the *primary part* or "*stator*", which is driven by our hand with a speed *v*. The, so-called, *secondary part* or "*rotor*" is moving slightly behind the magnetic traveling field of the primary part but with the same speed *v*. Since it moves synchronously with the magnetic travelling field we call this motor a *synchronous motor*.

The name *reluctance* means, that the reluctance of the magnetic circuit, which the magnetic flux is closed through, will change as the flux is moving on, because of finite length of a secondary part. Due to this change the magnetic force is produced that acts on the “rotor”. If the secondary part would be very (infinitely) long there would be no magnetic force F_x that drive the “rotor”. There will be only attractive force F_y (see Fig.1).

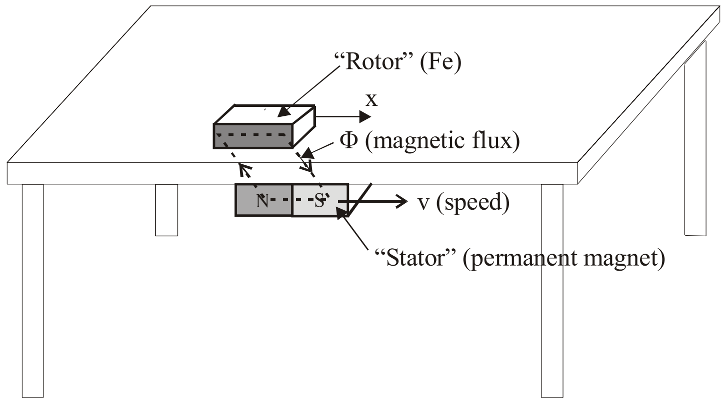

Fig.1 Linear reluctance motor formed by moving permanent magnet and a piece of solid iron

1.2. Permanent magnet (linear) synchronous motor

The secondary part can be in a form of permanent magnet as shown in Fig.2. In this case the magnetic force acting on the “rotor” is an effect of interaction of two magnetic fields: one produced by the “stator” and another one by the “rotor”. The “rotor” moves here synchronously with the magnetic field of the “stator”. Because of that, and, since the secondary part is in form of permanent magnet the motor is called a *permanent magnet synchronous motor.*

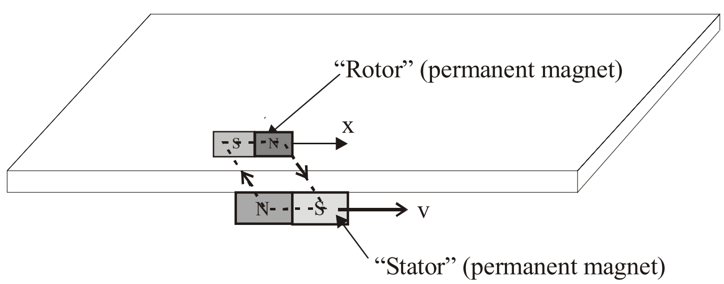

Fig.2 Linear permanent magnet synchronous motor formed by the moving “stator” permanent magnet that is pulling the “rotor” permanent magnet

So far we analyzed the motors where the primary magnetic field has been produced by the permanent magnet. This can be replaced by the electromagnet. Its coil may be supplied from the dc voltage source as it is shown in Fig.3. The operation of the motor does not differ from the previous motor and to get the magnetic flux moving we must push the electromagnet. The motor still can be called *permanent magnet linear synchronous motor*.

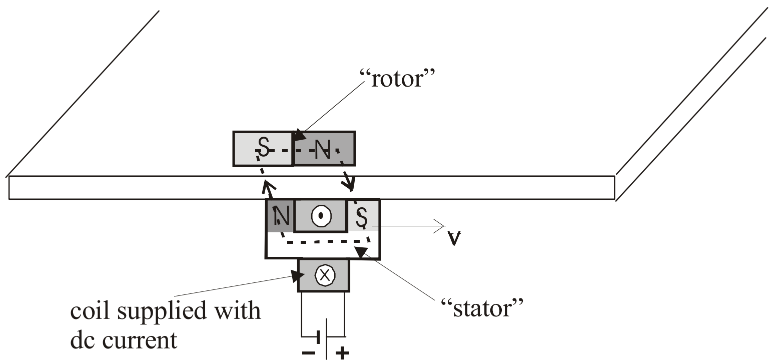

Fig.3 Permanent magnet (linear) synchronous motor: “stator” magnetic field is produced by the coil supplied from the dc source

1.3 Synchronous (linear) motor

The permanent magnet of the secondary part can be replaced by the electromagnet as shown in Fig.4. In this case the motor is called *synchronous linear motor*.

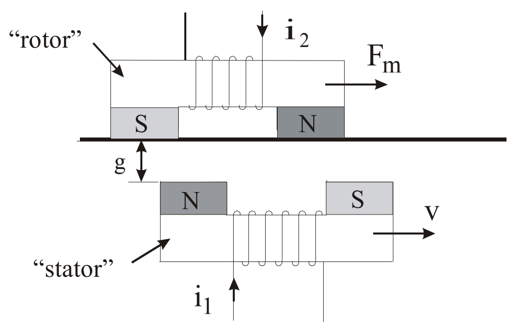

Fig.4 Linear synchronous motor: “stator” and “rotor” are electromagnets

The rotating counterpart of the linear motion structure shown in Fig.4 is the machine (in this particular case – motor) shown in Fig.5. The stator (primary part) electromagnets, supplied by the current i_1 represent a rotating magnetic field that rotates with angular speed ω_m. The rotor electromagnet supplied by the current i_2 is driven by the torque T, slightly behind the stator magnetic field by an angle δ but with the same speed ω_m.

The magnetic force, which acts on the secondary magnet, depends on the mutual displacement of both parts. If this displacement is expresses in terms of the angle δ as shown in Fig.6, then the force F_m changes, practically, sinusoidally as shown in the graph in Fig.7.

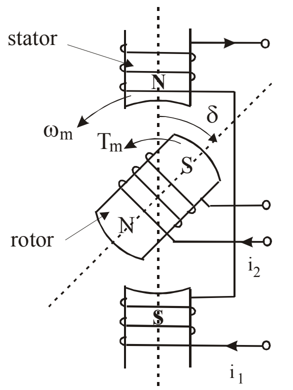

Fig.5 Rotary synchronous motor

The angle δ is counted from the primary part to the secondary part. Therefore it is negative. The maximum negative value the force reaches at angle (*-90*^{0}). The force can be expressed by the function:

$$F_m = F_{\max} \sin(\delta) \tag{1}$$

or expressed in terms of Cartesian's coordinate:

$$F_m = F_{\max} \sin\left(\frac{\pi}{\tau} x\right) \tag{2}$$

where τ is the magnetic pole-pitch (see Fig.4).

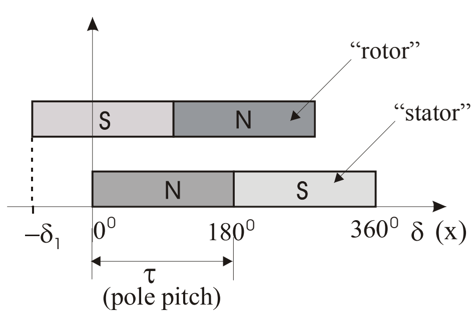

Fig.6 Linear synchronous motor

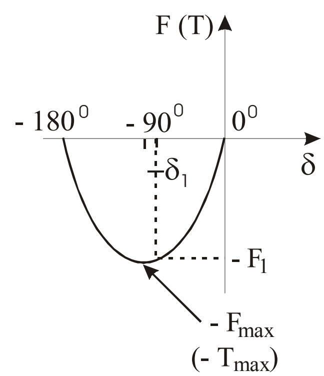

Fig.7 Force (torque) – power angle characteristic of linear (rotary) synchronous motor

In case of cylindrical structure shown in Fig.5 the rotor rotates synchronously with the stator magnetic flux due to the torque T acting on the rotor. Its value depends sinusoidaly on the angle δ as shown in Fig.7.

2. Force and torque in electric machines

In electric motors the electric energy is converted into mechanical energy. In electric generator the process of energy conversion is reversed: a mechanical energy is converted into electrical energy. In both cases the magnetic field (magnetic flux Φ - see Fig.8) is the medium in the *electromechanical conversion* process.

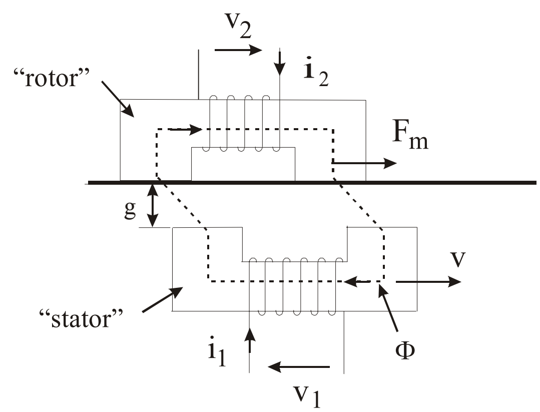

Fig.8 Illustration to electromechanical energy conversion

Look at Fig.8. In the linear synchronous motor the *electric energy* is delivered to the system through the "stator" and "rotor" winding terminals called *electrical ports*. This energy is converted to the *energy of magnetic field*, which is next converted into *mechanical energy.* The entire process of energy conversion is shown schematically in Fig.9. For the generator the process is reversed: the mechanical energy is delivered to the rotor through a rotor shaft and due to the magnetic flux generated by the rotor current it is converted into electrical energy leaving the system through the stator winding terminals (Fig.10).

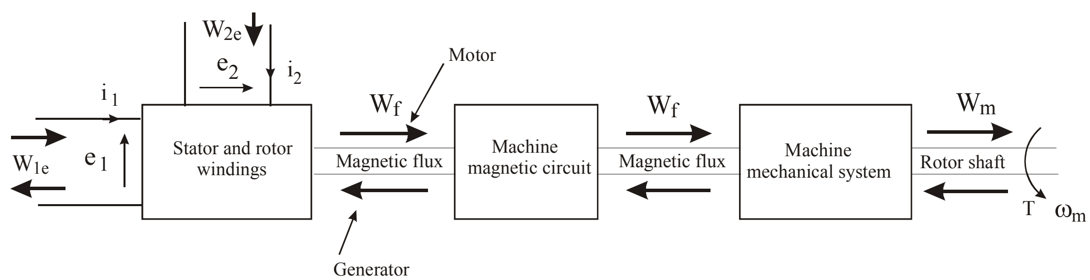

Fig.9 Diagram of electromechanical energy conversion; no power losses are included

During the process of energy conversion the power losses dissipate in the system. In the rotor and stator windings a part of electrical energy is converted into heat due to Ohmic power losses in the winding resistance. In the rotor and stator cores, part of field energy is lost. In the mechanical part of the system part of mechanical energy is lost as heat in bearings and due to the friction between the rotating rotor and the air (windage losses). The process of energy conversion with inclusion of power losses is shown in Fig.11. These power losses are converted into the *heat energy*.

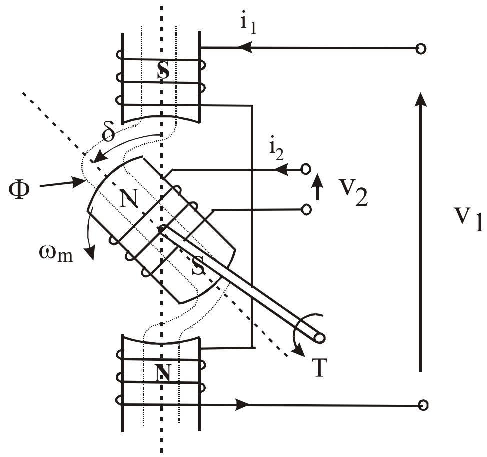

Fig.10 Illustration to electromechanical energy conversion in rotary synchronous generator

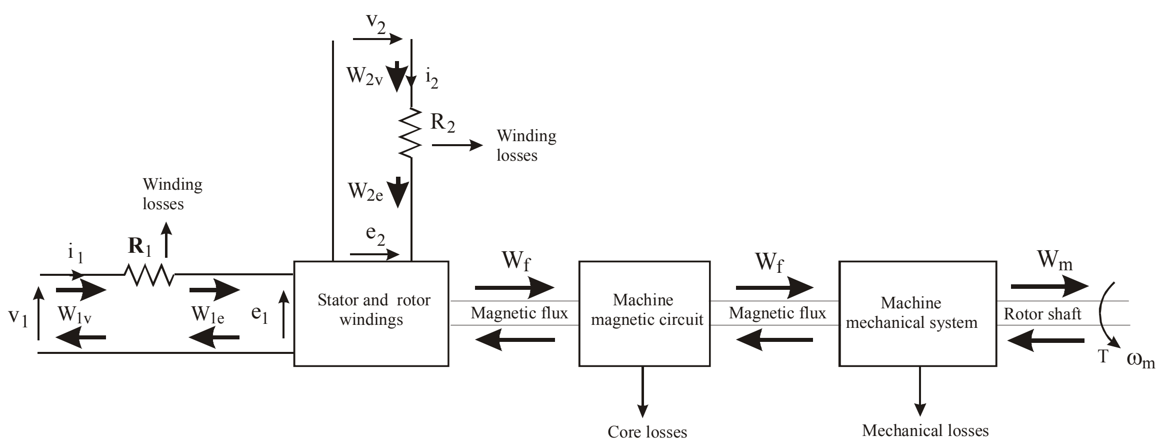

Fig.11 Diagram of electromechanical energy conversion with inclusion of power losses

2.1 Field energy

In both: motor and generator the field energy is converted either into electric or mechanical energy. In permanent magnet machine the magnetic flux is generated by the magnet and in case of electromagnet the magnetic field is generated by the current.

To determine the magnetic field energy stored in the motor let us consider the electromagnetic structure shown in Fig.12 consisting of primary part, which does not move and the secondary part, that can move and which does not have the winding. Let assume that at present the secondary part does not move. Suppose we increase now the current in the primary winding from *0* to i_1. The magnetic flux will rise from *0* to Φ_1 as shown in Fig.13. We can express the magnetic flux as the flux linkage $\lambda = N \cdot \Phi$, which is the product of a number of winding turns and the magnetic flux. In case of the real magnetic circuit the λ-i curve is not linear due to the saturation of the iron core. For the linear magnetic circuit the λ-i characteristic is a straight line as shown in Fig.13.b. This straight line is described by the equation:

$$\lambda = L \cdot i \tag{3}$$

where L is a current i coefficient known as winding inductance. If we differentiate both sides of the above equation assuming L = *const*, we will obtain the equation for the voltage e induced in the winding:

$$e = \frac{d\lambda}{dt} = L\frac{di}{dt} \tag{4}$$

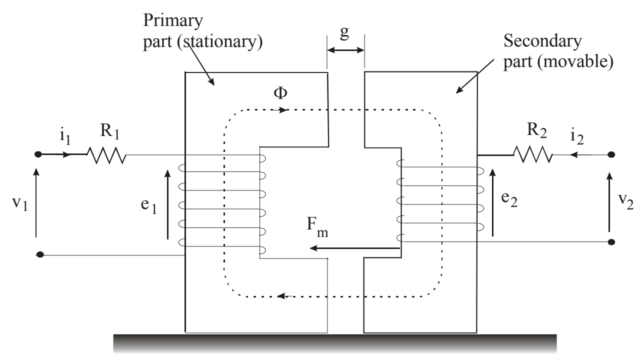

Fig.12 Illustration to derivation of formula for field energy

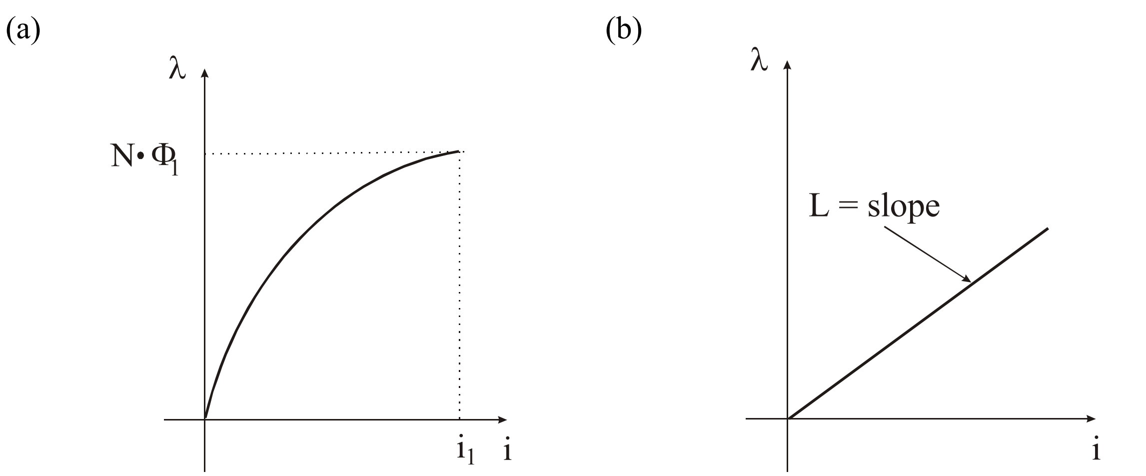

Fig.13 Magnetic linkage-current characteristic for: (a) – nonlinear system, (b) – for linear system

The electric power is equal:

$$p_e = e \cdot i = L\frac{di}{dt}i \tag{5}$$

Since the relation between the power and energy is

$$\frac{dW_e}{dt} = p_e \tag{6}$$

The increment of electric energy:

$$dW_e = p_e \cdot dt = e \cdot i \cdot dt = L \cdot i \cdot di \tag{7}$$

In this particular case this energy is a part of the total electric energy delivered to the winding (see Fig.11):

$$dW_v = p_v \cdot dt \tag{8}$$

where:

$$p_v = v \cdot i = R \cdot i^2 + e \cdot i \tag{9}$$

Thus W_e is equal to the magnetic field energy stored in the magnetic flux:

$$W_e = W_f \tag{10}$$

If the power losses in all elements of the system are ignored and the secondary part is moving, then, during the differential time interval *dt* the increment of electrical energy dW_e is equal to the sum:

$$dW_e = dW_f + dW_m \tag{11}$$

where dW_m is the increment of mechanical energy equal to mechanical work done during the time *dt* by the moving secondary part.

If the losses cannot be neglected they can be dealt with separately. They do not contribute to the energy conversion process.

When the flux linkage is increased from zero to λ_1 by means of increase of current from *0* to i_1, the energy stored in the field is (Fig.14):

$$W_f = \int_0^{\lambda_1} i\, d\lambda \tag{12}$$

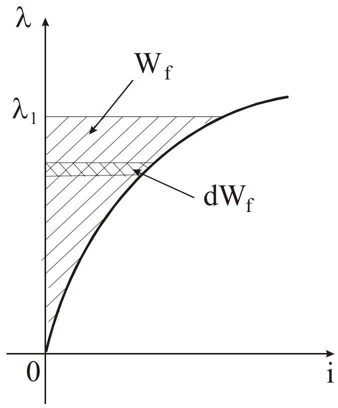

Fig.14 Field energy on λ-i characteristic

Suppose the air gap of the system in Fig.12 increases. The λ-i characteristic will become more flat and straight (see Fig.15). To maintain the same magnetic flux (flux linkage) greater current should flow in the winding and consequently greater energy is stored in the magnetic circuit (Fig.16). Since the volume of magnetic core remained unchanged the increase of field energy occurred in the air-gap.

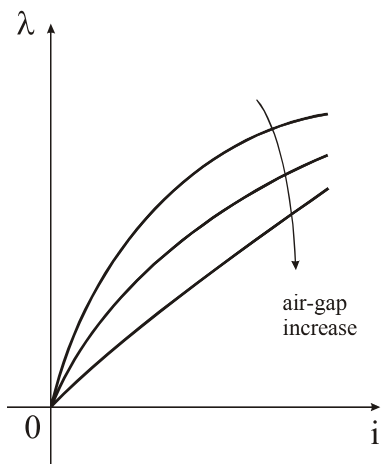

Fig.15 λ-I characteristics for various air-gaps in the machine

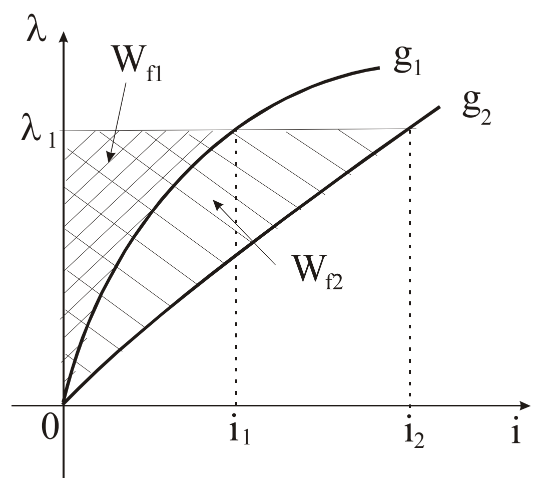

Fig.16 Field energy in the machine with different air-gap

The energy stored in the field can be expressed in terms of other quantities, e.g. magnetic flux density B in the air gap g. To find flux density B for a given current i in the winding we will use the equivalent magnetic circuit of the system shown in Fig.17. This circuit does not differ from electric circuit for a dc current and the analogy between the electric and magnetic quantities are shown in Table 1.

Table 1

Electric circuit	Magnetic circuit
Electromofive force (emf) E [V] Current I [A]	Magnetomotive (mmf) $F_m = I \cdot N$ [A·turns] Magnetic flux Φ [Wb]

Resistance of conductor $$R = \frac{l_w}{A_w \gamma} \; [\Omega]$$ where: l_w – length of wire [m], A_w – cross-section area of the wire [m^2], γ – conductivity [..] Ohm's law: $$i = \frac{E}{R}$$	Magnetic resistance (reluctance) of magnetic circuit $$R_m = \frac{l_m}{A_m \mu} \; [1/H] \qquad (13)$$ where: l_m – length of magnetic circuit [m], A_m – cross-section area of magnetic circuit [m^2] μ – magnetic permeability [H/m] Ohm's law: $$\Phi = \frac{F_m}{R_m} \qquad (14)$$

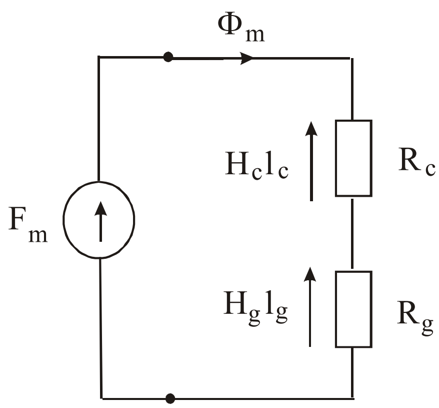

Fig.17 Equivalent magnetic circuit of the electromagnetic system shown in Fig.12

Magnetic flux Φ is a product:

$$\Phi = B \cdot A_m \tag{15}$$

of B [T] – magnetic flux density, and A_m [m^2] . For the linear magnetic circuit a flux density is equal

$$B = H \cdot \mu \tag{16}$$

where H [A/m] – is the magnetic field intensity in the magnetic circuit.

The Ohm's law for magnetic circuit written in other form is:

$$F_m = \Phi \cdot R_m \tag{17}$$

Inserting (13), (15) and (16) into (17) we obtain:

$$
\begin{aligned}
I \cdot N &= B \cdot A_m \frac{l_m}{A_m \mu} \\
&= H \cdot \mu \frac{l_m}{\mu} \\
&= H \cdot l_m
\end{aligned} \tag{18}
$$

where $H \cdot l_m$ – is the magnetic voltage drop across reluctance of the magnetic circuit.

Consider now the electromagnetic system shown in Fig.12 with its magnetic equivalent circuit in Fig.17. Let

H_c - magnetic intensity in the core

H_g - magnetic intensity in the air gap

l_c – total length of the magnetic core

l_g – length of the air-gaps

Then

$$N \cdot i_1 = H_c l_c + H_g l_g \tag{19}$$

The flux linkage:

$$\lambda = N \cdot \Phi = N \cdot A_m \cdot B \tag{20}$$

From Eqs.12, 19 and 20:

$$W_f = \int \frac{H_c l_c + H_g l_g}{N} N \cdot A_m \cdot dl \tag{21}$$

For the air-gap

$$H_g = \frac{B}{\mu_0} \tag{22}$$

where μ_0 – is the magnetic permeability of the vacuum (air-gap) equal to $4\pi 10^{-7}$ [H/m]

From Eqs.21 and 22

$$
\begin{aligned}
W_f &= \int \left(H_c l_c + \frac{B}{\mu_0} l_g \right) A_m \cdot dB \\
&= \int \left(H_c dB \cdot A_m \cdot l_c + \frac{B}{\mu_0} dB \cdot A_m \cdot l_g \right) \\
&= \int H_c dB \cdot V_c + \frac{B^2}{2\mu_0} \cdot V_g \\
&= w_{fc} \cdot V_c + w_{fg} \cdot V_g \\
&= W_{fc} + W_{fg}
\end{aligned} \tag{23}
$$

where:

$w_{fc} = \int H_c dB$ - is the energy density in the magnetic core

$w_{fg} = \dfrac{B^2}{2\mu_0}$ - is the energy density in the air-gap

V_c – is the volume of the magnetic core
V_g – is the volume of the air-gap
W_{fc} – is the energy in the magnetic core
W_{fg} – is the energy in the air-gap

For a linear magnetic core:

$$H_c = \frac{B_c}{\mu_c} \tag{24}$$

therefore

$$\begin{aligned} W_{fc} &= \int \frac{B_c}{\mu_c} dB_c \cdot V_c \\ &= \frac{B_c^2}{2\mu_c} \cdot V_c \end{aligned} \tag{25}$$

Looking at Eqs. 23 and 25 we see that the field energy is inversely proportional to the permeability μ and straight proportional to the volume V. If we have the electromechanical system shown in Fig.12, in which the same flux density is in both: iron cores and air-gap (the same flux Φ and the same cross-section area are for both parts), the magnetic energy is stored mainly in the air-gap, since the core permeability is equal to $\mu_c = \mu_o \mu_r$, and the relative permeability for unsaturated iron is $\mu_r > 1000$.

2.2 Co-energy

To calculate the attractive magnetic force acting on the movable part we will introduce the quantity called co-energy. It is defined as:

$$W_f^{'} = \int_0^{i_1} \lambda \cdot di \tag{26}$$

It does not have any physical significance. Co-energy and energy of the system is shown in Fig.18. From Fig.18

$$W_f^{'} + W_f = \lambda \cdot i \tag{27}$$

If λ-i characteristic is nonlinear: $W_f^{'} > W_f$ (Fig.19 – curve g_1), but if λ-i characteristic is linear (straight line g_2): $W_f^{'} = W_f$. If the air-gap increases from g_1 to g_2 and the current remains unchanged the co-energy will decrease as shown in Fig.19.

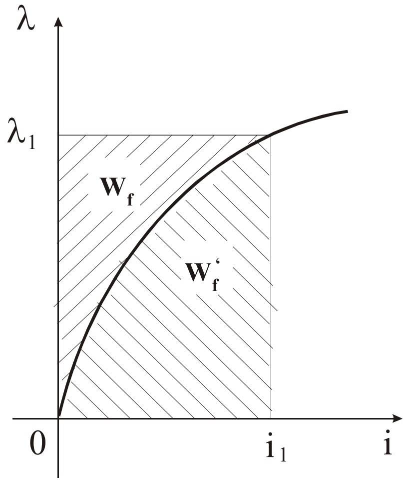

Fig.18 Field energy W_f and field co-energy W_f'

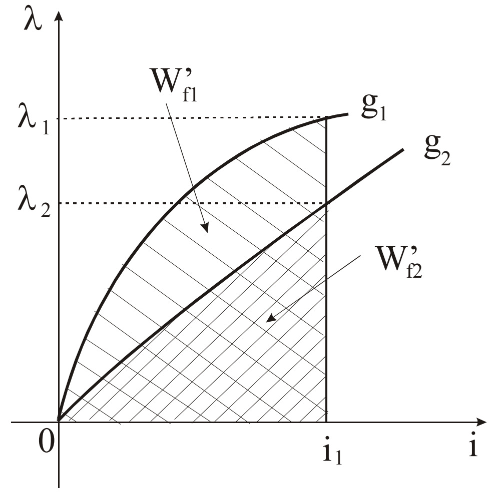

Fig.19 Field co-energy for two different values of air-gap in the system

2.3 Mechanical energy and forces

Let us consider the system in Fig.20. Let the secondary part moves from one position ($x = x_1$) to another position ($x = x_2$). The λ-i characteristics of the system for these two positions are shown in Fig.21. If the secondary part has moved slowly the current, equal to $i = v / R$ remains the same at both positions in the steady state because the coil resistance does not change and the voltage is set to be constant.

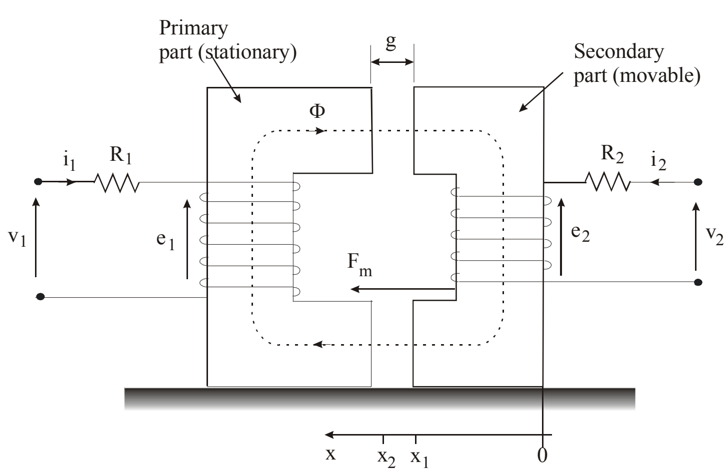

Fig.20 Electromechanical system with movable and stationary parts

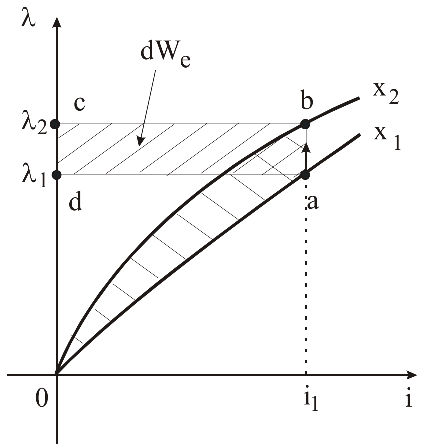

Fig.21 Illustration to the magnetic force derivation

The operation point has moved upward from point $a \rightarrow b$. During the motion:

$$dW_e = \int e \cdot i \cdot dt = \int_{\lambda_1}^{\lambda_2} i \cdot d\lambda = area(abcd) \tag{28}$$

the increment of electric energy has been sent to the system. The field energy has changed by the increment

$$dW_f = area(0bc - 0ad) \tag{29}$$

The mechanical energy

$$\begin{aligned} dW_m &= dW_e - dW_f \\ &= area(abcd) + area(0ad) - area(0bc) \\ &= area(0ab) \end{aligned} \tag{30}$$

is equal to the mechanical work done during the motion of the secondary part and it is represented by the shaded area in Fig.21. This shaded area can be seen also as the increase in the co-energy:

$$dW_m = dW_f^{'} \tag{31}$$

Since:

$$dW_m = f_m dx \tag{32}$$

the force f_m that is causing differential displacement is:

$$f_m = \left. \frac{\partial W_f^{'}(i,x)}{\partial x} \right|_{i=const} \tag{33}$$

2.3.1 Force in linear system

Let us consider again the system shown in Fig.20. The reluctance of the magnetic core path can be ignored due to the high value of μ_c (see Eq.13) and the λ-i characteristic is assumed to be linear. The coil inductance L_1 depends on the reluctance of the magnetic circuit. From Eqs.3 and 14 we obtain

$$L = \frac{N \cdot F_m}{R_m i} \tag{34}$$

From Eqs.13 and 34, after transformation we obtain:

$$L = \frac{N^2 \mu A_m}{g} \tag{35}$$

That means that inductance L depends on length of the air-gap, so it is the function of x co-ordinate (see Fig.20). Thus for the idealized system:

$$\lambda = L(x) i \tag{36}$$

where $L(x)$ changes its value with the gap length.
Since the field co-energy is:

$$W_f' = \int_0^i \lambda \cdot di \tag{37}$$

after inserting Eq.36 for λ we obtain:

$$\begin{aligned} W_f' &= \int_0^i L(x) i \cdot di \\ &= \frac{1}{2} L(x) i^2 \end{aligned} \tag{38}$$

The magnetic force acting on the secondary part we obtain from Eqs.33 and 38:

$$\begin{aligned} f_m &= \frac{\partial}{\partial x}\left(\frac{1}{2} L(x) i^2\right)\Bigg|_{i=const} \\ &= \frac{1}{2} i^2 \frac{dL(x)}{dx} \end{aligned} \tag{39}$$

For a linear system the field energy is equal to the co-energy (Fig.19 – line g_2), thus:

$$W_f = W_f^{'} = \frac{1}{2} L(x) i^2 \tag{40}$$

Force f_m can be expressed also in terms of magnetic flux density in the air-gap B_g. If we assume that H_c is negligible (due to high permeability μ_c of the core), then for mechanical system in Fig.20 we obtain from Eq.19:

$$Ni = H_g 2g = \frac{B_g}{\mu_0} 2g \tag{41}$$

and

$$i = \frac{B_g}{N\mu_0} 2g \tag{42}$$

From Eqs.35, 38 and 42 we obtain:

$$W_f^{'} = \frac{B_g^2}{2\mu_0} \cdot A_g \cdot 2g \tag{43}$$

The above expression can be obtained also from the field energy. For the linear magnetic circuit $W_f^{'} = W_f$, therefore from Eq.23 (for negligible magnetic energy stored in the core):

$$\begin{aligned} W_f^{'} &= \frac{B_g^2}{2\mu_0} \cdot V_g \\ &= \frac{B_g^2}{2\mu_0} \cdot A_g \cdot 2g \end{aligned} \tag{44}$$

where A_g is the cross-section area of the air-gap.

From Eqs.33 and 43 the force acting on the secondary part is:

$$\begin{aligned} f_m &= \frac{\partial}{\partial g}\left(\frac{B_g^2}{2\mu_0} \cdot A_g \cdot 2g \right) \\ &= \frac{B_g^2}{2\mu_0} \cdot 2A_g \end{aligned} \tag{45}$$

It means that the magnetic force is proportional to the magnetic flux density in square.

The magnetic pressure F_m that is often used is calculated as the force per unit area of air-gap. Thus from Eq.45 we have (the cross-section area of the air-gap is $2A_g$):

$$F_m = \frac{B_g^2}{2\mu_0} \tag{46}$$

The expression 45 can be used to calculate the force in all electromechanical devices where the magnetic flux density in the air-gap is known. So it can also be applied to determine the driving force in all these primitive linear motors shown in Fgs.1 - 4. Let us consider the reluctance linear "motor" shown in Fig.22. The magnetic force acting on the "rotor" (and the same force, but in opposite direction, acts on the "stator") is expressed by Eq.45. The driving force f_x acting on the "rotor" is the tangential (to x axis) component that can be determined from f_m if we know the angle β or the mutual displacement Δx between two motor parts.

If we have the reluctance motor with the primary part as an electromagnet (Fig.23) we can use formula 39 to determine the linear force f_x acting on the secondary part. The data that we have to know is the current i flowing in the "stator" winding and the inductance $L(x)$ expressed as the function of x coordinate. If the "rotor" is infinitely long in x direction as in Fig.24, then $L = const.$ with respect to x direction and according to Eq.23 the force $f_x = 0$ despite very strong attractive force f_y acting on the "rotor".

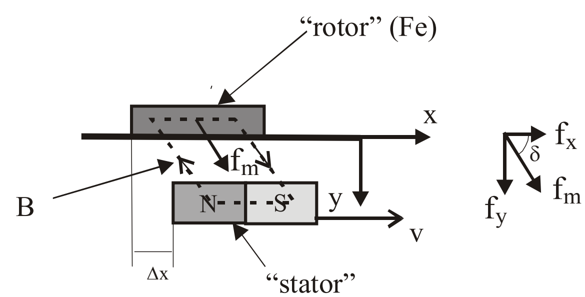

Fig.22 Force components in linear reluctance motor

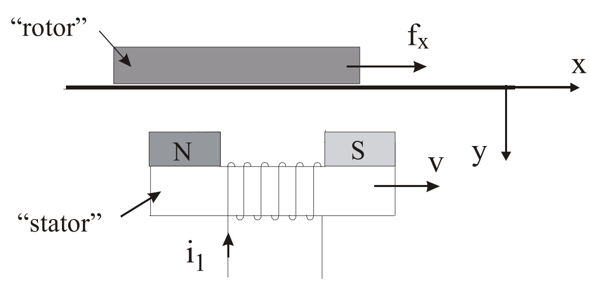

Fig.23 Force component F_x produced in the linear reluctance motor

The Eqns.39 and 45 for force was derived for the system with the coil wound on the primary part only. We derive now the force equation for the linear system with the coil wound also on the secondary part (Fig.20) as it is in synchronous linear motors shown in Fig.8. To do this let us assume again that the secondary part does not move and the field energy is equal to the electric energy:

$$dW_f = dW_e = e_1 i_1 dt + e_2 i_2 dt$$
$$= i_1 d\lambda_1 + i_2 d\lambda_2 \tag{47}$$

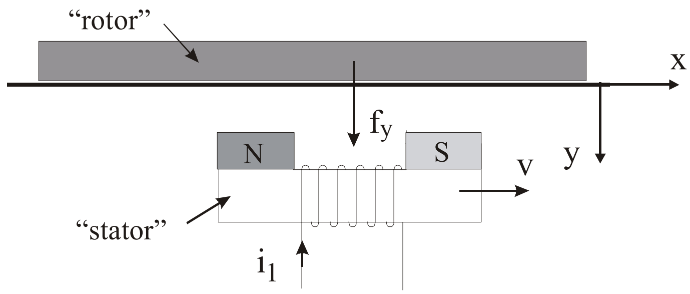

Fig.24 No driving force in the motor with infinitely (very) long “rotor”

In the linear system (the λ-i characteristic is represented by the straight line; see Fig13.b) the flux linkages can be expressed in terms of inductances that are constant:

$$\begin{aligned} \lambda_1 &= L_{11} i_{11} + L_{12} i_2 \\ \lambda_2 &= L_{21} i_1 + L_{22} i_2 \end{aligned} \tag{48}$$

where: L_{11} – is the self inductance of the excitation winding
L_{22} – is the self inductance of the moveable part winding
L_{12} and L_{21} – are the mutual inductances between two windings

From Eqs.33 and 34:

$$\begin{aligned} dW_f &= i_1 d\left(L_{11} i_1 + L_{12} i_2\right) + i_2 d\left(L_{22} i_2 + L_{21} i_1\right) \\ &= L_{11} i_1 di_1 + L_{22} i_2 di_2 + L_{12} d\left(i_1 i_2\right) \end{aligned} \tag{49}$$

The field energy is equal to the field co-energy for the linear systems. Thus we have:

$$\begin{aligned} W_f = W_f^{'} &= L_{11} \int_0^{i_1} i_1 di_1 + L_{22} \int_0^{i_2} i_2 di_2 + L_{12} \int_0^{i_1 i_2} d\left(i_1 i_2\right) \\ &= \frac{1}{2} L_{11} i_1^2 + \frac{1}{2} L_{22} i_2^2 + L_{12} i_1 i_2 \end{aligned} \tag{50}$$

In the system analyzed here the inductances depend on the value of an air-gap. It means, that they are functions of position x of secondary part. According to Eq.33 the force developed by the linear system:

$$
\begin{aligned}
f_m &= \left.\frac{\partial W_f^{'}(i,x)}{\partial x}\right|_{i=const} \\
&= \frac{1}{2} i_1^2 \frac{dL_{11}}{dx} + \frac{1}{2} i_2^2 \frac{dL_{22}}{dx} + i_1 i_2 \frac{dL_{12}}{dx}
\end{aligned}
\tag{51}
$$

The first two terms are the two force components known as reluctance forces. The third one is called an electromagnetic force. This force exists even if self inductances do not depend on x co-ordinate, it is if there are no two first components. This type of situation exists in the system shown in Fig.25, where either two or one of the part is infinitely (very) long. When the secondary moves with respect to the primary, self inductances of the coils remain unchanged, and only the mutual magnetic coupling (mutual inductance) changes.

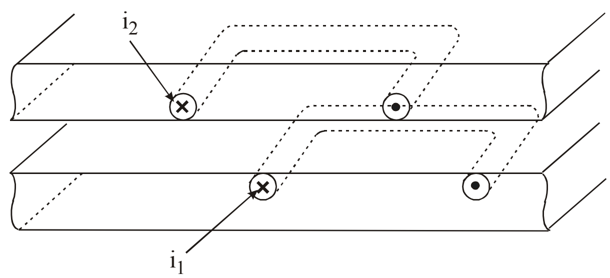

Fig.25 Coils in the infinitely long "stator" and "rotor" cores

Equilibrium equations

To analyze transients in the windings and dynamic behavior of the motors we have to write *equilibrium equations* for *electrical ports* and *mechanical port*. The equilibrium equations for electrical ports are terminal *voltage equations* for both primary and secondary windings written on the basis of the second (voltage) Kirhhoff's law applied to equivalent circuit of both windings shown in Fig.26. Equation for mechanical port is the *equation of motion* for mechanical system of the motor shown in Fig.27, and written in accordance with the Newton's Law of Motion. The equations written for the electromechanical system shown in Fig.20 are as follows:

- for electrical ports:

$$v_1 = R_1 i_1 + \frac{\partial \lambda_1}{\partial t} \tag{52}$$

$$v_2 = R_2 i_2 + \frac{\partial \lambda_2}{\partial t} \tag{53}$$

- for mechanical port:

$$f_m = M\frac{d^2x}{dt^2} + D\frac{dx}{dt} + f_L \tag{54}$$

where:

M – is mass of the movable part

D – friction coefficient of the movable secondary part

f_L – load force

In the above voltage equations the terminal voltages are equal to the voltage drops across the winding resistances and the voltages e_1 and e_2 (see Fig.26) induced in the windings by the fluxes linked with them. In the motion equation the electromagnetic force developed by the motor is equal to the inertia force, friction force and load force.

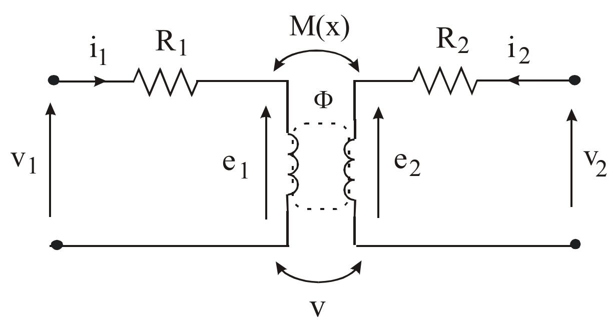

Fig.26 Equivalent circuit of the electromechanical system in Fig.20

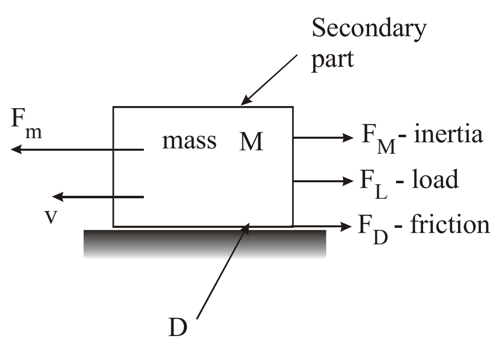

Fig.27 Equivalent mechanical system of the electromechanical system in Fig.20

The derivatives:

$$\begin{aligned}\frac{\partial\lambda_1}{\partial t} = e_1 &= \frac{\partial\left(L_{11}(x)i_1\right)}{\partial t} + \frac{\partial\left(L_{12}(x)i_2\right)}{\partial t} \\ &= L_{11}(x)\frac{di_1}{dt} + i_1\frac{dL_{11}(x)}{dx}\frac{dx}{dt} + L_{12}(x)\frac{di_2}{dt} + i_2\frac{dL_{12}(x)}{dx}\frac{dx}{dt}\end{aligned} \tag{55}$$

$$\frac{\partial\lambda_2}{\partial t} = e_2 = \frac{\partial\left(L_{22}\left(x\right)i_2\right)}{\partial t} + \frac{\partial\left(L_{21}\left(x\right)i_1\right)}{\partial t}$$
$$= L_{22}\left(x\right)\frac{di_2}{dt} + i_2\frac{dL_{22}\left(x\right)}{dx}\frac{dx}{dt} + L_{21}\left(x\right)\frac{di_1}{dt} + i_1\frac{dL_{21}\left(x\right)}{dx}\frac{dx}{dt} \qquad (56)$$

Examples of electromechanical devices with a linear (oscillating) motion:

- ***transformer with moving coil*** (for secondary voltage variation) (Fig.28)

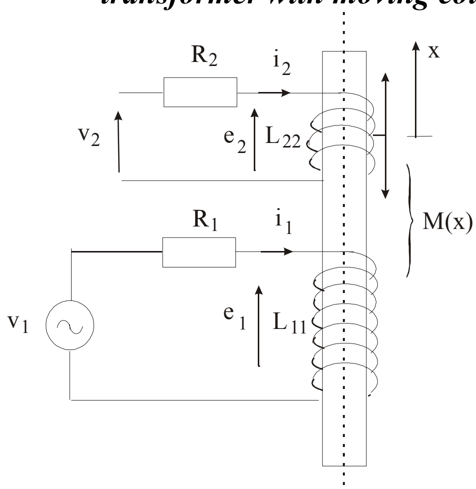

Fig.28 Equivalent circuit of transformer

Voltage equations:

$$v_1 = R_1 i_1 + \frac{\partial\lambda_1}{\partial t}$$

$$v_2 = R_2 i_2 + \frac{\partial\lambda_2}{\partial t}$$

since $\lambda_1 = L_{11}i_{11} + L_{12}i_2$ *and* $\lambda_2 = L_{21}i_1 + L_{22}i_2$

and $L_{11},\ L_{22}$ *and* $L_{12} = L_{21} = M(x)$ are constant in time

$$v_1 = R_1 i_1 + L_{11}\frac{di_1}{dt} + M\left(x\right)\frac{di_2}{dt}, \text{ or } v_1 = R_1 i_1 + e_1$$

$$v_2 = R_2 i_2 + L_{22}\frac{di_2}{dt} + M\left(x\right)\frac{di_1}{dt}, \text{ or } v_2 = R_2 i_2 + e_2$$

Changing the distance between the coils we influence M(x) and we change the voltage e_2 and v_2.

- ***jumping ring*** (Fig.29)

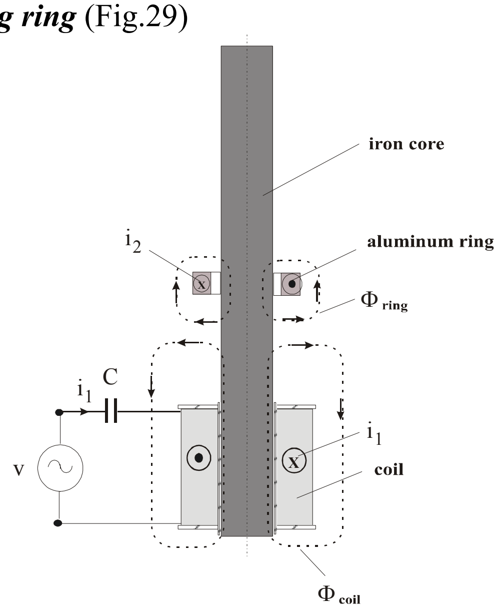

Fig.29 Jumping ring

<u>Idea of operation:</u> Magnetic flux of the coil Φ_{coil} induces the voltage E_2 (and the current i_2) in the ring. The current i_2 in the ring contributes to the flux Φ_{ring}. This flux opposes the flux Φ_{coil} according to phasor diagram in Fig.30.b (the inductances L_{11} and E_{22} are assumed to be much greater than the resistances R_1 and R_2). As the result the ring is repelled out of the coil.

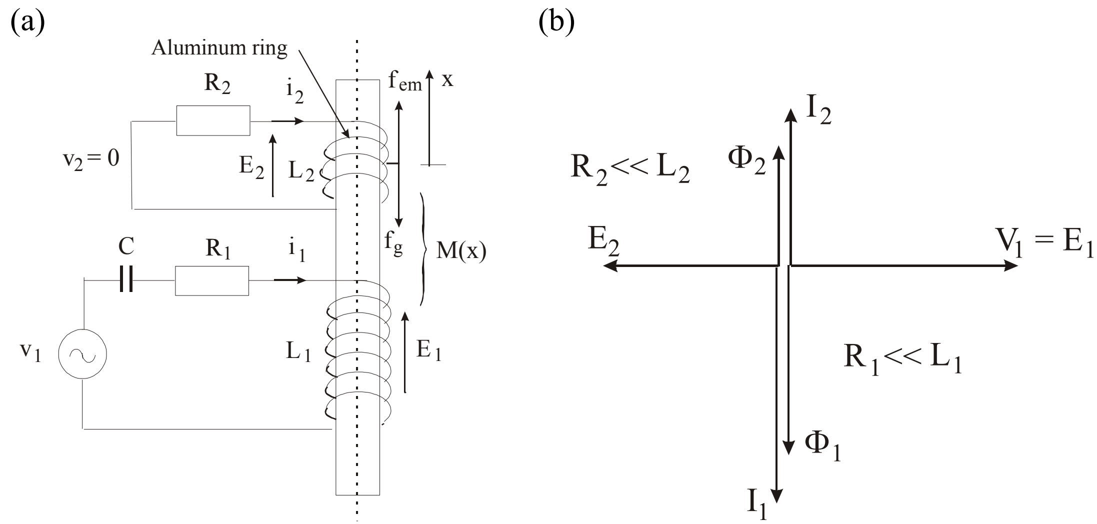

Fig.30 Equivalent circuit of the *jumping ring* (a), and its phasor diagram (b)

When the distance between the coil and ring changes, then the resultant inductance of the coil is changing too. If the capacitor C has the value that brings the coil circuit into resonance when the ring is at lowest position, then the heavy coil current contributes to the strong force that repels the ring upwards. When the ring is far from the coil the resonance disappears, consequently the lifting force goes to zero and the ring falls down. There the resonance occurs again and the ring is repelled again. The resultant effect is the jumping ring. The equivalent circuit of the device with jumping ring is shown in Fig.30.a.

- ***induction linear oscillator*** (Fig.31)

The phenomenon described above is applied in induction oscillation motor shown schematically in Fig.31. Two coils are placed at the ends of the ferromagnetic bar. Both

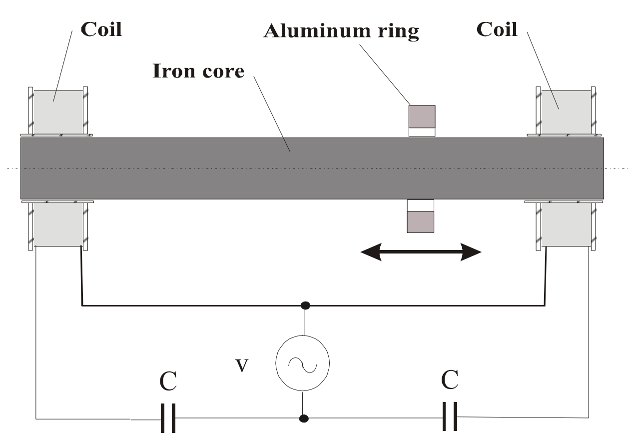

Fig.31 Scheme of induction linear oscillator

are connected in parallel to the AC source via capacitors. If the ring placed between them is close to one of the coil then the resonance occur in the coil and strong repulsion produced force throws the ring to the other coil. When the ring appears close to this coil the resonance of this coil will contributes to the repulsion of the ring to the opposite coil. Consequently, the ring oscillates between the coils.

- ***Reluctance linear oscillator*** (Fig.32)

The ferromagnetic bar loosely placed in the coil is suspended by the attractive magnetic force produced by the coil magnetic flux (Fig.32.a). When the bar moves in the coil the coil inductance changes its value. If the capacitor connected in series to the coil is chosen to put the circuit into resonance when the bar is at lower position (see Fig.32.b), then, due to the heavy resonance current, a strong magnetic force pulls the bar upwards into the coil. There the circuit is out of resonance and due to the small current (and no magnetic force at the center of the coil) the bar falls down. When it comes at the end of the coil the resonance appears again, pulling the bar upwards. In consequence the bar will oscillate in vertical position.

If the coil is placed in horizontal position as in Fig.33, then the resonance current appear at both ends. The bar will oscillate in horizontal position. To increase the efficiency of the oscillator *end-rings* are placed at both ends (see Fig.34). At the end of the bar track the kinetic energy of the moving bar is not lost but converted into the potential energy of the spring and next returned to the bar. This reluctance oscillator is more efficient than the induction oscillator because there are no current losses in the moving part.

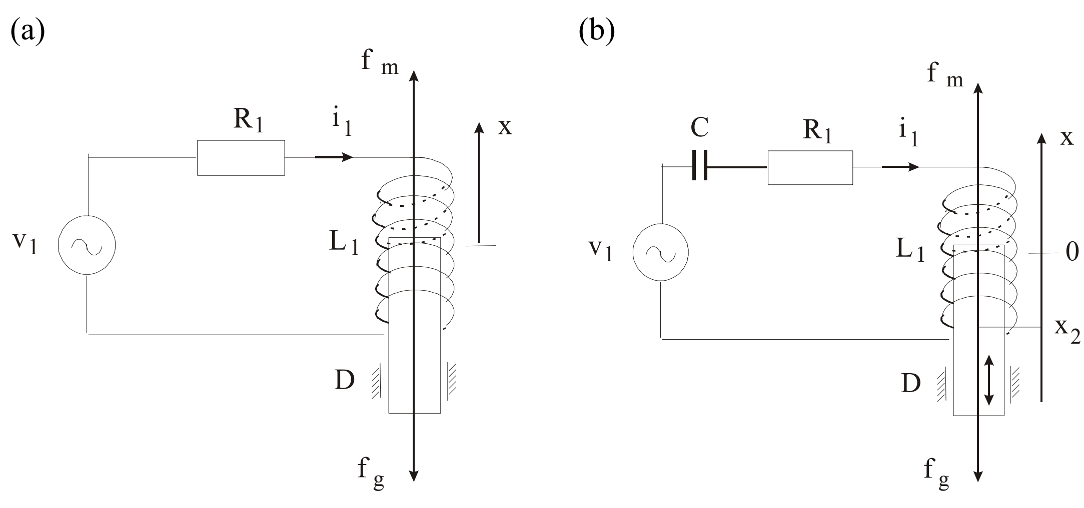

Fig.32 Circuit diagrams: (a) ferromagnetic bar suspended by the magnetic field, (b) electromagnetic device with jumping ferromagnetic bar

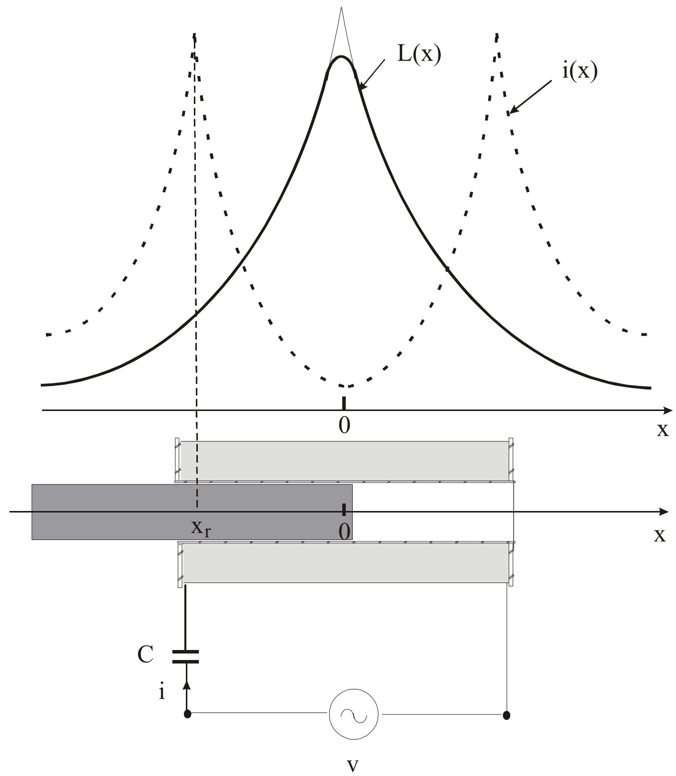

Fig.33 Scheme of the reluctance linear oscillator with the current and coil inductance characteristics

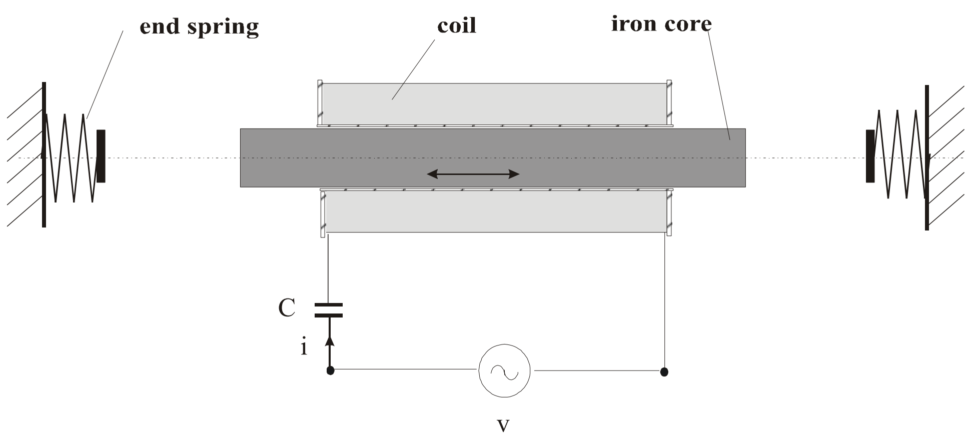

Fig.34 Reluctance linear oscillator with end springs

2.3.2 Torque in rotating machines

Let us consider the rotating electromagnetic system shown in Fig.35, in which the stator possesses winding 1 and the rotor – winding 2. The field co-energy is expressed by Eqn.50. The torque developed by the motor is

$$T = \left.\frac{\partial W_f^{'}(i,\theta)}{\partial \theta}\right|_{i=const} \tag{57}$$

From Eqs.50 and 57:

$$T = \frac{1}{2} i_1^2 \frac{dL_{11}}{d\theta} + \frac{1}{2} i_2^2 \frac{dL_{22}}{d\theta} + i_1 i_2 \frac{dL_{12}}{d\theta} \tag{58}$$

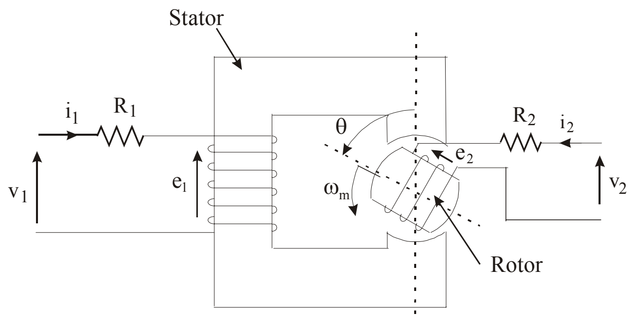

Fig.35 Rotary motor with salient pole stator and rotor

The first two terms are reluctance torques and the third one is an electromagnetic torque. For the motor with the round rotor (Fig.36) there is no torque represented by the first term, since the self inductance of the stator L_{11} does not depend on the position of the torque (the magnetic flux generated by the first winding does not change as the rotor rotates). Therefore the torque of this motor is described by the following equation:

$$T = \frac{1}{2} i_2^2 \frac{dL_{22}}{d\theta} + i_1 i_2 \frac{dL_{12}}{d\theta} \tag{59}$$

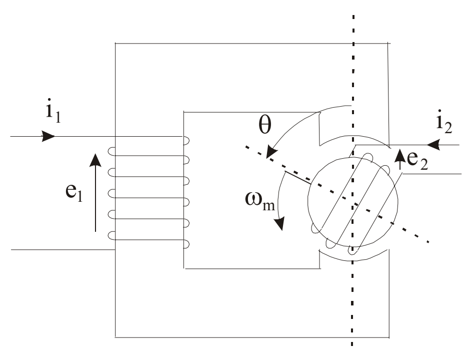

Fig.36 Motor with the salient pole stator and round rotor

Suppose the stator has cylindrical structure and the rotor has salient magnetic poles as shown in Fig.37. In such a motor, known as synchronous motor with salient poles, the self inductance of the rotor winding L_{22} does not change as the rotor rotates. Therefore the torque equation takes the form:

$$T = \frac{1}{2} i_1^2 \frac{dL_{11}}{d\theta} + i_1 i_2 \frac{dL_{12}}{d\theta} \tag{60}$$

The first term is the reluctance torque, and the second one is known as a synchronous torque.

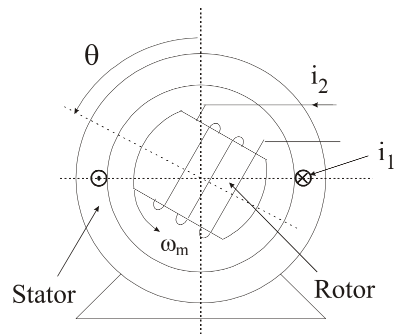

Fig.37 Motor with round stator and salient pole rotor

Machines with cylindrical stator and rotor

A scheme of cylindrical machine is shown in Fig.38. The self inductances are constant and therefore no reluctance torque is produced. The torque developed by the motor is

$$T = i_1 i_2 \frac{dL_{12}}{d\theta} \tag{61}$$

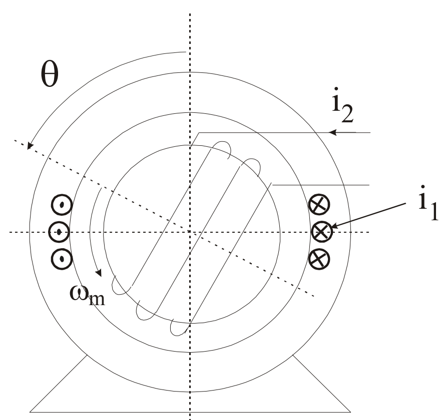

Fig.38 Motor with cylindrical stator and rotor

Let the mutual inductance changes sinusoidally:

$$L_{12} = M\cos\theta \tag{62}$$

where: M – is the peak value of mutual inductance
θ – is the angle between the magnetic axis of the stator and rotor windings.

Let the currents in the two windings be:

$$i_1 = I_{1m}\cos\omega_1 t \tag{63}$$

$$i_2 = I_{2m}\cos\left(\omega_2 t + \alpha\right) \tag{64}$$

The position of the rotor with respect to the stator depends on rotor speed and is:

$$\theta = \omega_m t + \delta \tag{65}$$

where: ω_m – is the angular velocity of the rotor
δ – is the rotor position at $t = 0$

From Eqns.61, 62, 63, 64 and 56 we have:

$$\begin{aligned} T &= -I_{1m}I_{2m}M\cos\omega_1 t\cos\left(\omega_2 t+\alpha\right)\sin\left(\omega_m t+\delta\right) \\ &= -\frac{I_{1m}I_{2m}M}{4}\left\{\sin\left[\left(\omega_m+\left(\omega_1+\omega_2\right)\right)t+\alpha+\delta\right]\right. \\ &\quad +\sin\left[\left(\omega_m-\left(\omega_1+\omega_2\right)\right)t-\alpha+\delta\right] \\ &\quad +\sin\left[\left(\omega_m+\left(\omega_1-\omega_2\right)\right)t-\alpha+\delta\right] \\ &\quad \left.+\sin\left[\left(\omega_m-\left(\omega_1-\omega_2\right)\right)t+\alpha+\delta\right]\right. \end{aligned} \tag{66}$$

The torque is the sum of four components, which vary sinusoidally with time. Therefore the average value of each component is zero unless the coefficients of t are zero. Thus the average torque will be nonzero if:

$$\omega_m = \pm\left(\omega_1 \pm \omega_2\right) \tag{67}$$

The machine will develop average torque if it rotates in either direction at a speed that is equal to the sum or difference of the angular frequencies $\omega = 2\pi f$ of the stator and the rotor currents

$$\left|\omega_m\right| = \left|\omega_1 \pm \omega_2\right| \tag{68}$$

There are two practical cases:

1) $\omega_2 = 0$, $\alpha = 0$, $\omega_m = \omega_1$: (single-phase synchronous machine). Rotor carries dc current, stator ac current.

For these conditions, from Eqn.66

$$T = -\frac{I_{1m} I_{2m} M}{2} \left\{ \sin\left(2\omega_1 t + \delta\right) + \sin\delta \right\} \tag{69}$$

The instantaneous torque is pulsating. It will be constant for poly-phase machine. To find the average torque from Eqn.69 we see that average of $\sin\left(2\omega_1 t + \delta\right)$ is zero. It means the average torque is:

$$T_{av} = -\frac{I_1 I_2 M}{2} \sin\delta \tag{70}$$

If $\omega_m = 0$ (at starting) the (single-phase) machine does not develop the average torque.

2) $\omega_m = \omega_1 - \omega_2$ (asynchronous single-phase motor). Both stator and the rotor carry ac currents at different frequencies and rotor speed $\omega_m \neq \omega_1$ and $\omega_m \neq \omega_2$

   From Eq.66

$$\begin{aligned} T = -\frac{I_{1m} I_{2m} M}{2} \Big\{ & \sin\left(2\omega_1 t + \alpha + \delta\right) + \sin\left(-2\omega_2 t - \alpha + \delta\right) \\ & + \sin\left(2\omega_1 t - 2\omega_2 t - \alpha + \delta\right) + \sin\left(\alpha + \delta\right) \Big\} \end{aligned} \tag{71}$$

The instantaneous torque is pulsating (it is constant in poly-phase motor). The average value of the torque is:

$$T_{av} = -\frac{I_{1m} I_{2m} M}{4} \sin\left(\alpha + \delta\right) \tag{72}$$

At $\omega_m = 0$ the average torque is zero. A single-phase machine should be brought to the speed different than 0 so it can produce an average torque.

This is the principle of operation of induction motor.

Equilibrium equations

Similar as for linear motor the equilibrium equations for rotating machine are as follows:

- for electrical ports: *voltage equations* (the same as Eqns.52 and 53).

$$v_1 = R_1 i_1 + \frac{\partial \lambda_1}{\partial t} \tag{73}$$

$$v_2 = R_2 i_2 + \frac{\partial \lambda_2}{\partial t} \tag{74}$$

The derivatives:

$$\begin{aligned}\frac{\partial \lambda_1}{\partial t} &= \frac{\partial\left(L_{11}(\theta) i_1\right)}{\partial t} + \frac{\partial\left(L_{12}(\theta) i_2\right)}{\partial t} \\ &= L_{11}(\theta)\frac{di_1}{dt} + i_1 \frac{dL_{11}(\theta)}{d\theta}\frac{d\theta}{dt} + L_{12}(\theta)\frac{di_2}{dt} + i_2 \frac{dL_{12}(\theta)}{d\theta}\frac{d\theta}{dt}\end{aligned} \tag{75}$$

$$\begin{aligned}\frac{\partial \lambda_2}{\partial t} &= \frac{\partial\left(L_{22}(\theta) i_2\right)}{\partial t} + \frac{\partial\left(L_{21}(\theta) i_1\right)}{\partial t} \\ &= L_{22}(\theta)\frac{di_2}{dt} + i_2 \frac{dL_{22}(\theta)}{d\theta}\frac{d\theta}{dt} + L_{21}(\theta)\frac{di_1}{dt} + i_1 \frac{dL_{21}(\theta)}{d\theta}\frac{d\theta}{dt}\end{aligned} \tag{76}$$

The term $\frac{\partial \theta}{\partial t} = \omega_m$ is the angular speed of the rotor. For the motor considered the mutual inductances $L_{12} = L_{21} = M$, thus:

$$e_1 = \frac{\partial \lambda_1}{\partial t} = L_{11}(\theta)\frac{di_1}{dt} + i_1 \frac{dL_{11}(\theta)}{d\theta}\omega_m + M(\theta)\frac{di_2}{dt} + i_2 \frac{dM(\theta)}{d\theta}\omega_m \tag{77}$$

$$e_2 = \frac{\partial \lambda_2}{\partial t} = L_{22}(\theta)\frac{di_2}{dt} + i_2 \frac{dL_{22}(\theta)}{d\theta}\omega_m + M(\theta)\frac{di_1}{dt} + i_1 \frac{dM(\theta)}{d\theta}\omega_m \tag{78}$$

The voltages induced in the windings (see Fig.36) form two groups. The first one contains the voltages:

$$e_{1t} = L_{11}(\theta)\frac{di_1}{dt} + M(\theta)\frac{di_2}{dt} \tag{79}$$

$$e_{2t} = L_{22}(\theta)\frac{di_2}{dt} + M(\theta)\frac{di_1}{dt} \tag{80}$$

induced due to the variation in time of the magnetic fluxes represented by the currents that generate them. These types of voltages are induced in transformers (index t). The second group:

$$e_{1r} = i_1 \frac{dL_{11}(\theta)}{d\theta}\omega_m + i_2 \frac{dM(\theta)}{d\theta}\omega_m \tag{81}$$

$$e_{2r} = i_2 \frac{dL_{22}(\theta)}{d\theta}\omega_m + i_1 \frac{dM(\theta)}{d\theta}\omega_m \tag{82}$$

are the voltages induced by the rotation of the rotor. In that number the first terms are the voltages induced by the saliency of the rotor and do not exist in the motor with cylindrical stator and the rotor (see Fig.38). The second terms are the voltages induced by the mutual rotation of the two windings.

- <u>for mechanical port</u>: *motion equation* (Fig.39)

$$T = J\frac{d^2\theta}{dt^2} + D\frac{d\theta}{dt} + T_s + T_l \qquad (83)$$

where:

J – moment of inertia of the rotor
D – friction coefficient of the rotor
T_l – load torque
T_s – torsional torque (see Fig.40 – two torques act in opposite directions and they twist the shafts by angle $\Delta\theta$), which is due to the rotor shaft elasticity and is given by:

$$T_s = K_s\left(\Delta\theta\right) \qquad (84)$$

where K_s is the torsional coefficient.

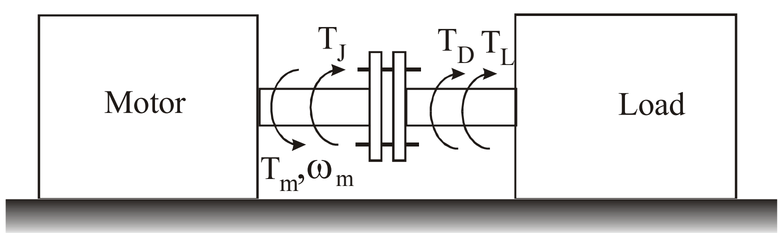

Fig.39 Equivalent diagram for mechanical system of the rotary motor: T_m – electromagnetic torque, T_J – inertia torque, T_D – friction torque, T_L – load torque

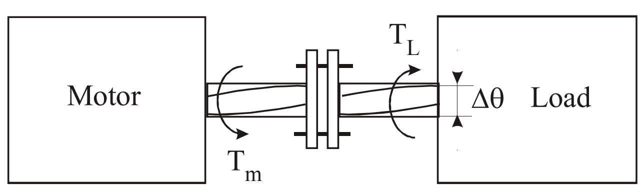

Fig.40 Twist (by the angle $\Delta\theta$) of the elastic shaft caused by electromagnetic torque of the motor T_m and load torque T_L acting in oposite direction